History-deterministic Timed Automata are Not Determinizable

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Kaiserslautern

History-Determinism (HD)

a restricted form of non-determinism which can be resolved on the fly

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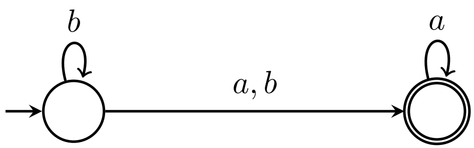

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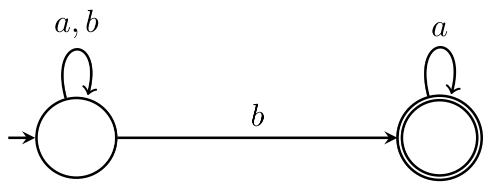

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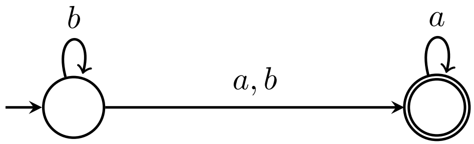

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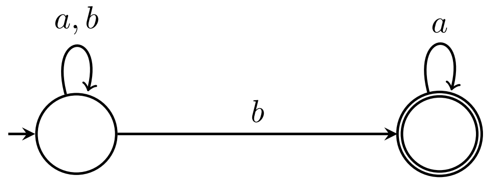

Non-deterministic

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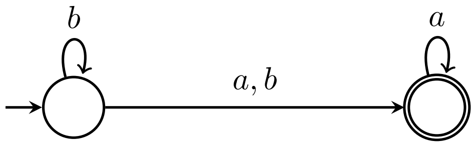

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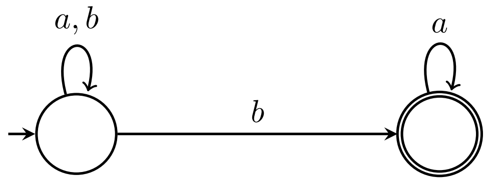

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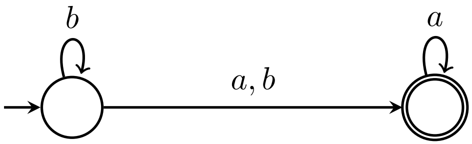

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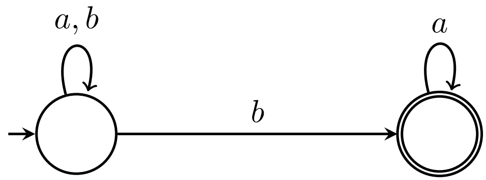

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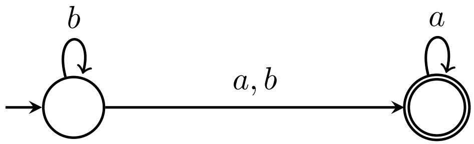

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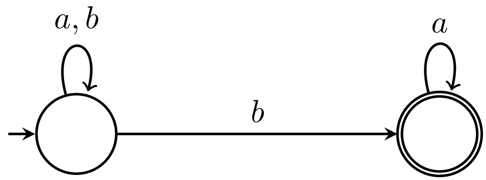

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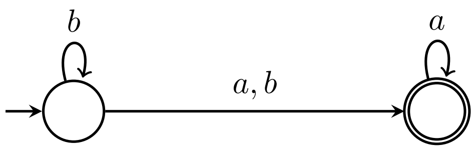

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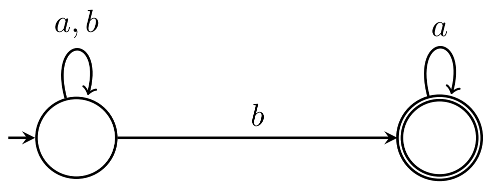

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D $\subseteq$ HD $\subseteq$ ND

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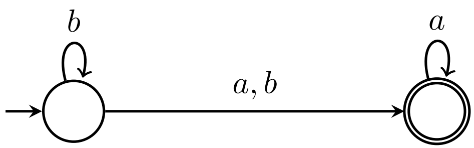

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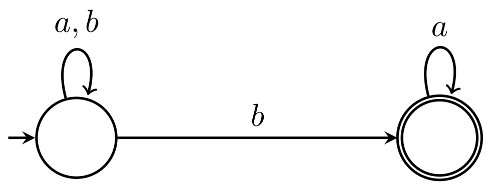

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D $\subseteq$ HD $\subseteq$ ND

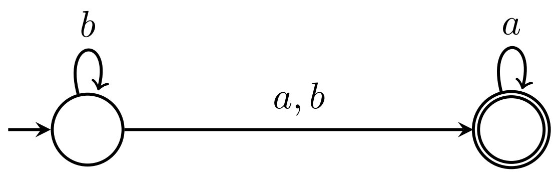
Are these containments strict?

Acceptance via Safety, Reachability, Büchi, co-Büchi, . . .

History-Determinism (HD)

A is HD if there exists a **resolver** function

$$r : \Sigma^* \times \Sigma \to T$$



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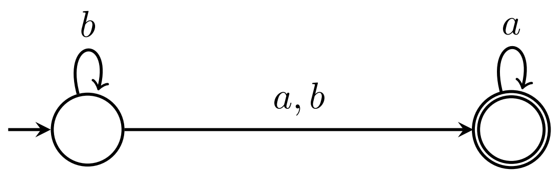

- Good-for-Trees [Kupferman, Safra, Vardi '96]
- Good-for-Games [Henzinger, Pieterman '06]
- Letter Games [Henzinger, Pieterman '06]
- History-deterministic [Colcombet '09]
- Fair Simulation [Henzinger, Lehtinen, Totzke '22]

History-Determinism (HD)

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ω-Automata

$$D\omega A = HD\omega A = ND\omega A$$

Pushdown Automata[Guha, Jecker, Lehtinen, Zimmermann '21]

$$DPA \subsetneq HDPA \subsetneq NDPA$$

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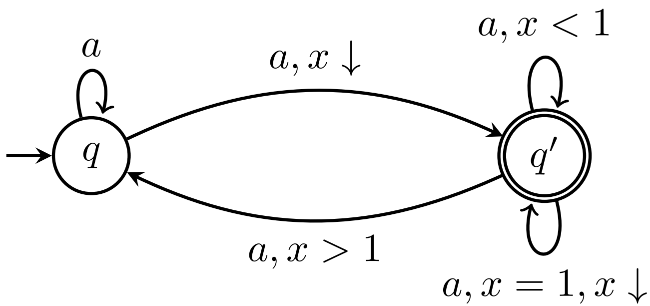
What about Timed Automata?

Timed Automata



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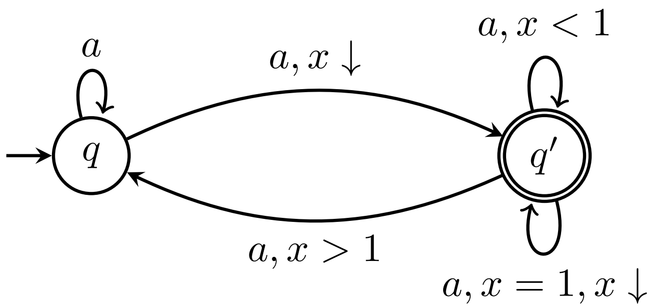
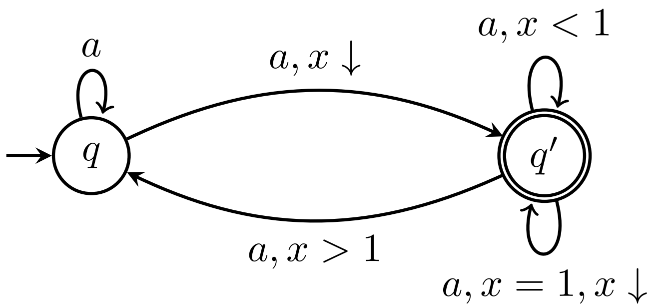


Co-Büchi Acceptance

Infinite non-zeno words

Letters:	*a*
Time:	0.5
Clock value *x*:	

Timed Automata



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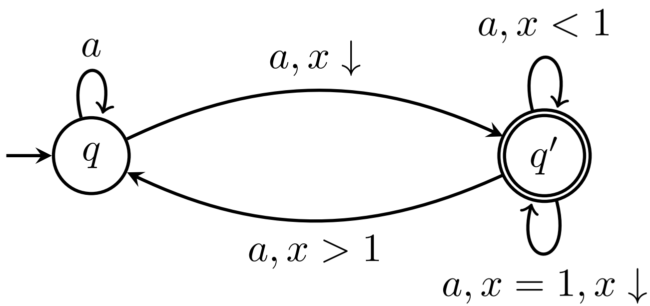

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Time:	0.5	1.5
Clock value x:	0.5	

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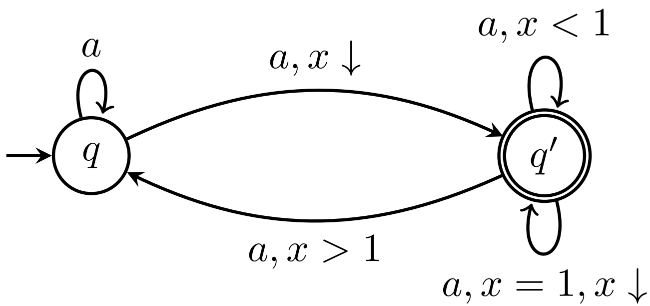

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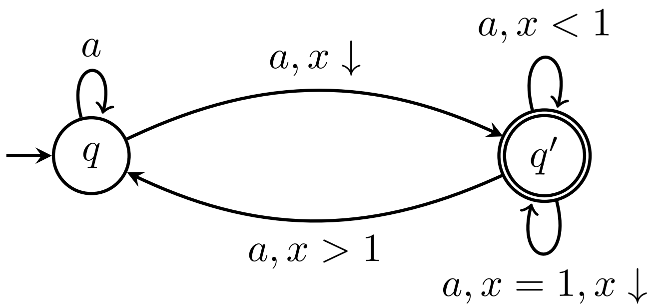

Co-Büchi Acceptance

Infinite non-zeno words

Letters:	*a*	*a*	*a*	*a*	*a*	*a*
Time:	0.5	1.5	2.1	2.6	3.1	4.1
Clock value x:	0.5	0	0.6	1.1	0	0

Timed Automata

Alur & Dill '94

Timed Automata

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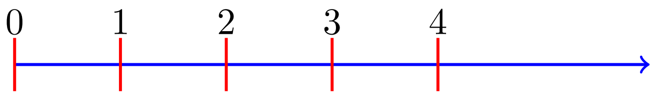
Region Abstraction

$x < 2, \quad x > 4, \quad x = 1$



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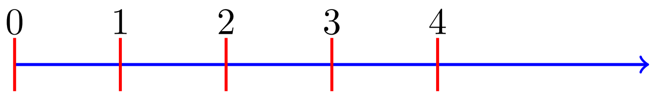

$0,\ (0,1),\ 1,\ \ldots,\ (4,\infty)$

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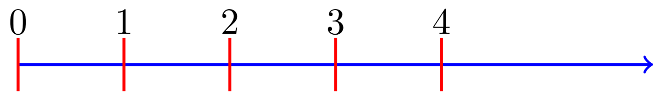
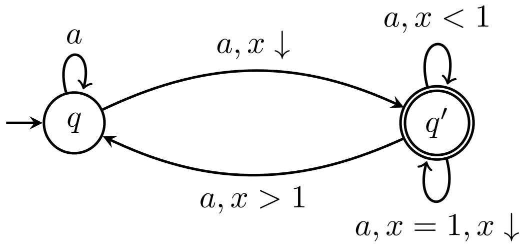


$$0,\ (0,1),\ 1,\ \ldots,\ (4,\infty)$$

- Same transitions are enabled from the same region
- Exponentially many regions in the number of clocks
- Used to solve Emptiness

History Deterministic Timed Automata



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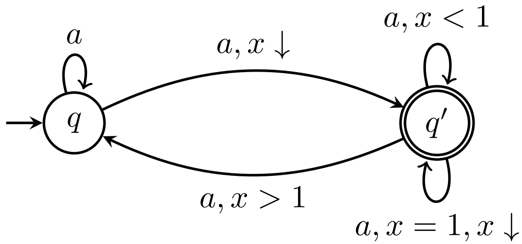

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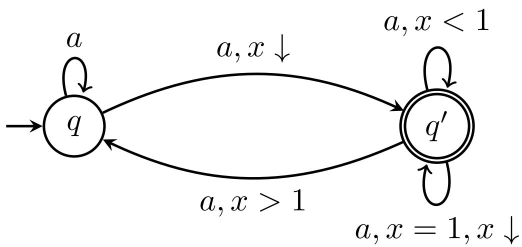

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Letters:	*a*	*a*	*a*	*a*	*a*	*a*
Time:	0.5	1.5	2.1	2.6	3.1	4.1

Resolver: Guess the oldest surviving fractional part with trailing *a* at unit distance

Expressiveness of HDTA

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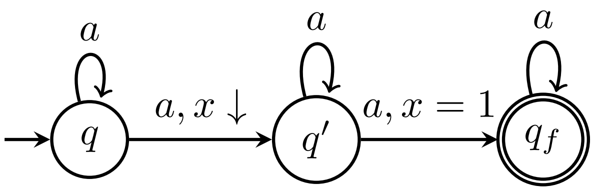

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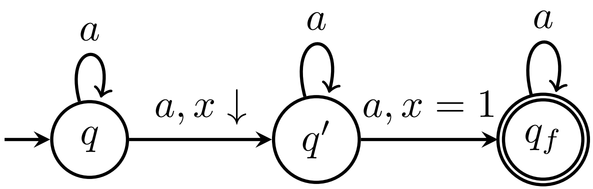

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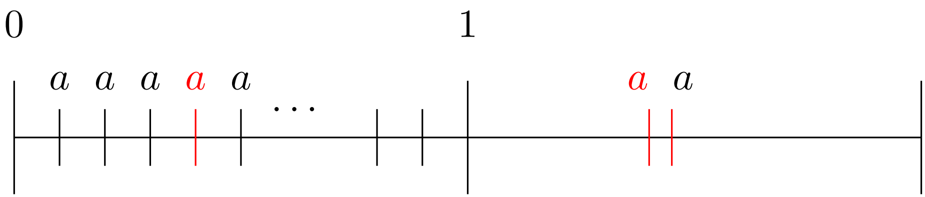
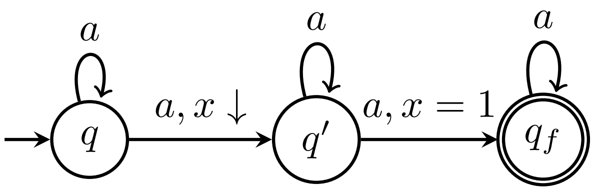


Claim: There is no equivalent HDTA with k clocks

Expressiveness of HDTA



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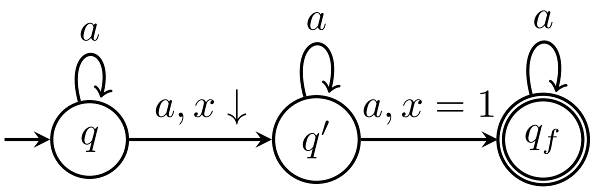

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Theorem (Henzinger, Lehtinen, Totzke '22)

Safety HDTA can be determinized

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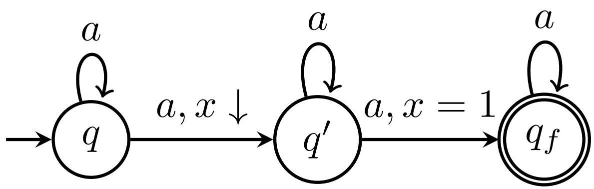

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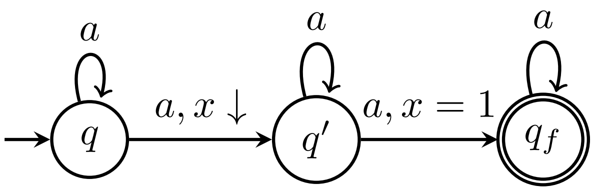

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Conjecture (Henzinger, Lehtinen, Totzke '22)

Co-Büchi HDTA can **not** be determinized

$$L = \{(a, t_0)(a, t_1)\ldots \mid \exists i \in \mathbb{N} \quad \forall n \in \mathbb{N}. \quad \text{there is an } a \text{ at time } t_i + n\}$$

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Theorem

L is not accepted by any parity DTA

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Theorem

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- Difficult to prove language is not deterministic
- Reset points based techniques [Herrmann] only apply in Büchi case

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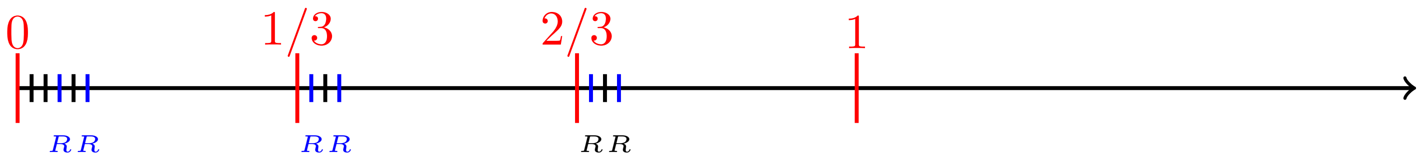

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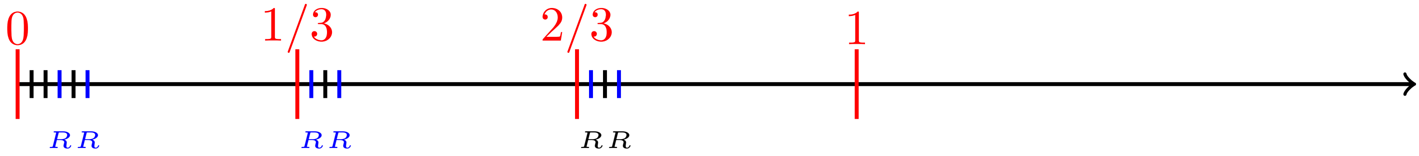

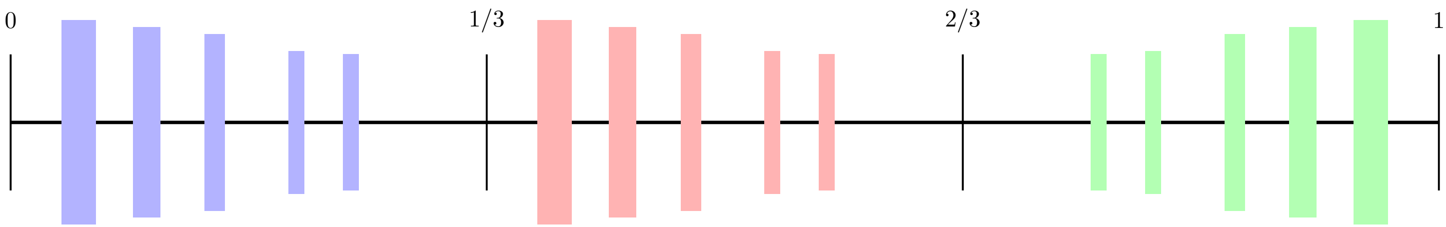

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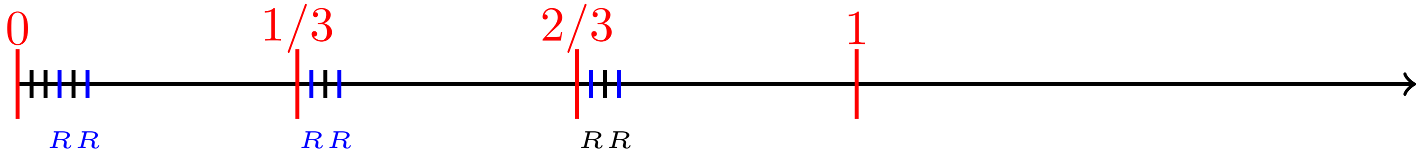

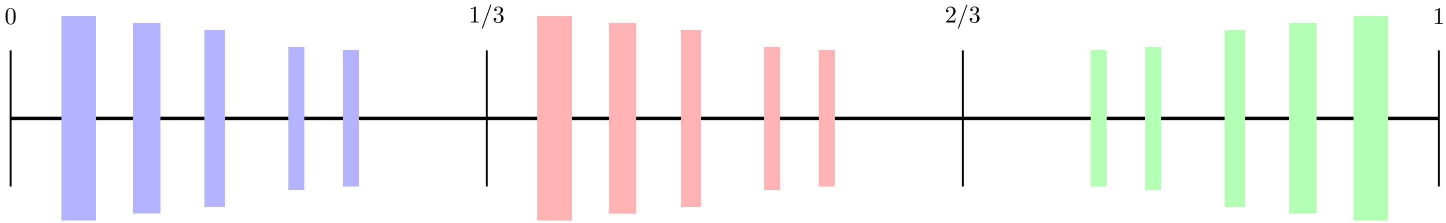

w : Pick ticks in blue, red and green blocks progressively

w' : Same as w but pick the same red block

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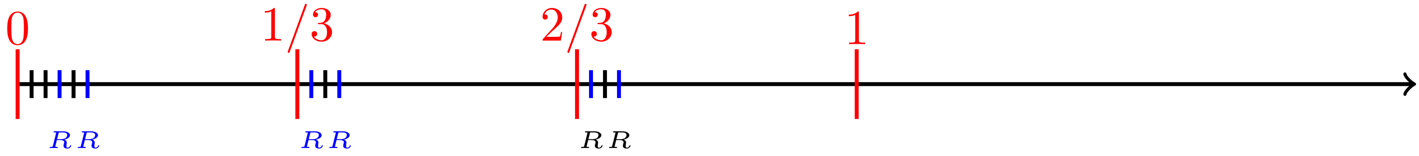

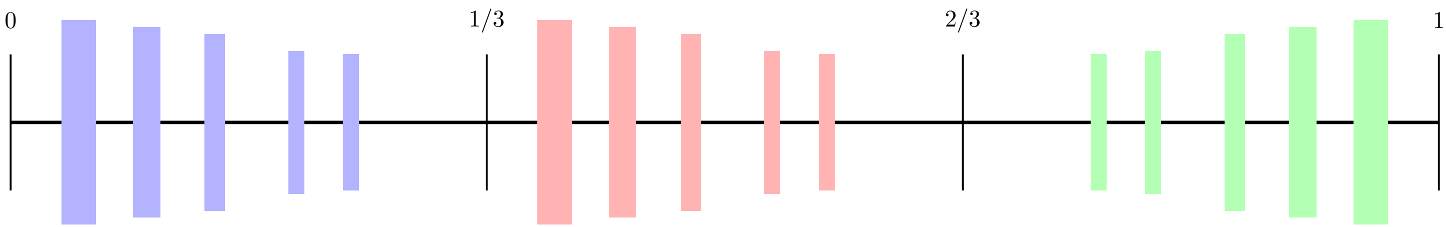

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w' : Same as w but pick the same red block

$w' \in L$, $w \notin L$

Both have the same runs

Summary and Future works

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Co-Büchi [Bose, Henzinger, Lehtinen, Schewe, Totzke '22]

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Reachability [Work in Progress]

$$DTA = HDTA \subsetneq NDTA$$

Büchi [Work in Progress]

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