A DYNAMIC PROGRAMMING ALGORITHM FOR A MAXIMUM s-CLIQUE SET ON TREES

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ROYAL HOLLOWAY UNIVERSITY OF LONDON

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Agenda

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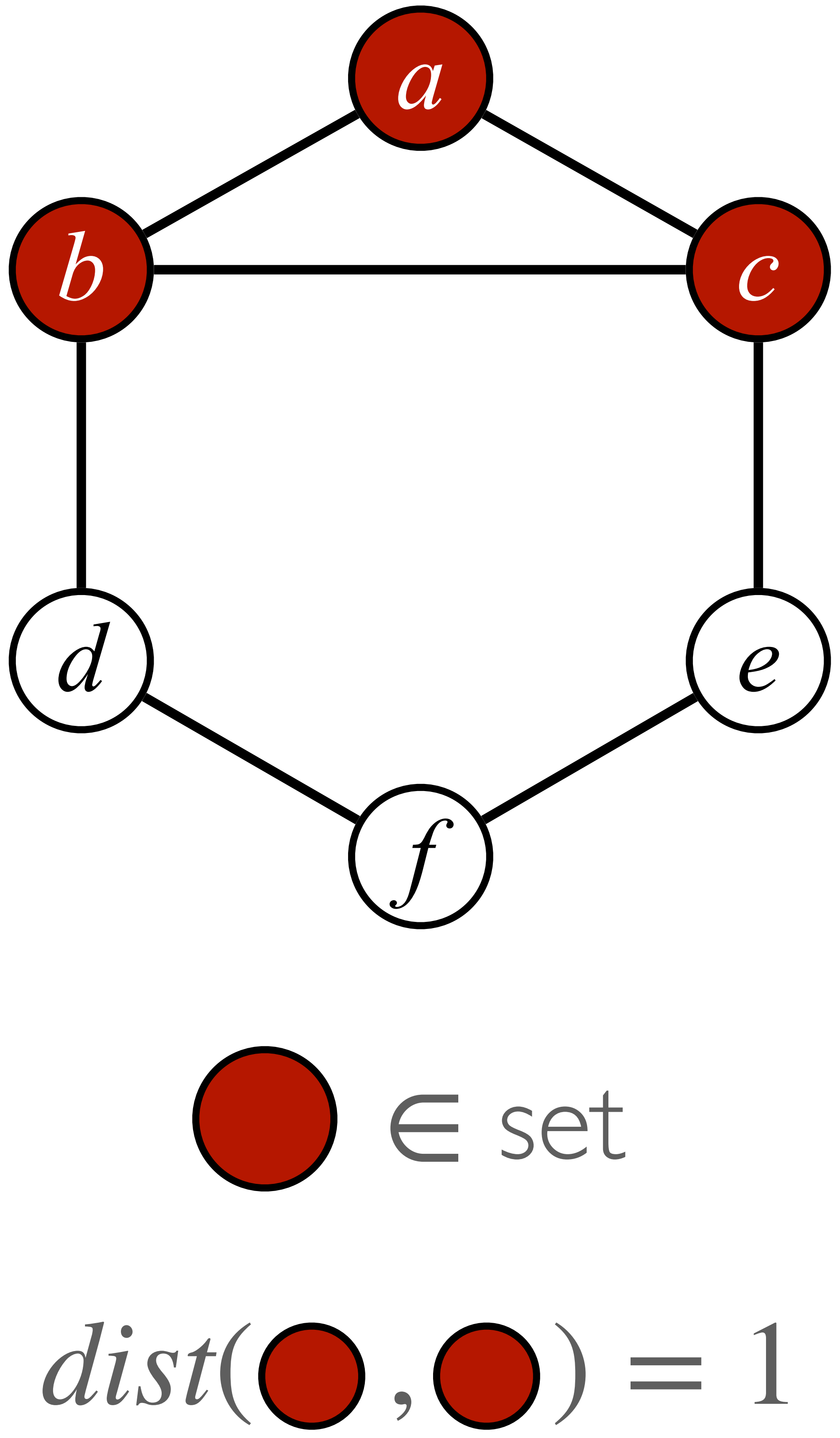
CLIQUE

Given a graph $G = (V_G, E_G)$ a ***clique*** is a subset of vertices $S \subseteq V_G$ such that

- Each pair of distinct vertices $u, v \in S$ are adjacent.
- In other words, a clique $G' = (S, E'_G)$ is a complete subgraph of G.
- The size of a clique is the number of vertices it contains.



DURING THE 50S

PSYCHOMETRIKA—VOL. 15, NO. 2
JUNE, 1950

CONNECTIVITY AND GENERALIZED CLIQUES IN SOCIOMETRIC GROUP STRUCTURE

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GRADUATE STUDENT, DEPARTMENT OF MATHEMATICS
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

By using the concepts of antimetry and n-chain it is possible to define and to investigate some properties of connectivity in a sociometric group. It is shown that the number of elements in a group, the number of antimetries, and the degree of connectivity must satisfy certain inequalities. Using the ideas of connectivity, a generalized concept of clique, called an n-clique, is introduced. n-cliques are shown to have a very close relationship to the existence of cliques in an artificial structure defined on the same set of elements, thus permitting the determination of n-cliques by means of the same simple matrix procedures used to obtain the clique structures. The presence of two or more m-cliques, where m is the number of elements in the group, is proved to mean an almost complete splitting of the group.

1. *Introduction*

This paper is devoted to extending the theoretical and practical mathematical results presented in an earlier paper (3). In that paper it was shown that certain elementary matrix operations permit, to some extent, an analysis of the simple type of psychological group structure which is often expressed by a sociometric diagram. Specifically, we envisage a finite set of m ($\geqslant 2$) elements $i, j, k, \cdots$ having a structure defined on them as follows: For any two elements i and j of the set there either exists or does not exist some *one type* of "directional" relationship *from* i *to* j. This one type of relationship on the group may assume various forms such as communication or friendship; and so to acquire generality in the discussion we shall speak of an *antimetry* from i to j. Then communication is merely a special type of antimetry. It is arbitrarily assumed that no antimetry can exist from an element i to itself. By "directional" is meant that knowing of the existence or non-existence of an antimetry from i to j does not give any information about the existence or non-existence of an antimetry from j to i. For example, if the form of the antimetry is "i communicates to j," then the specific knowledge that a can communicate to b does not tell us, in general, whether b can communicate to a.

Journal of Mathematical Sociology
1973 Vol. 3, pp 113-126

A GRAPH-THEORETIC DEFINITION OF A SOCIOMETRIC CLIQUE *

RICHARD D. ALBA

Columbia University

The intent of this paper is to provide a definition of a sociometric clique in the language of graph theory. This problem is viewed from two perspectives: maintaining fidelity to the intuitive notion of a clique; and providing adequate computational mechanics for large bodies of data. Luce's (1950) concept of an K-clique is used, but further qualifications are added. Two statistics or measures with associated probability distributions are defined for testing the adequacy of a subgraph which qualifies according to the definition.

1. INTRODUCTION

One of the factors which has most confused the discussion of sociometric clique identification in large bodies of data is the absence of a formal definition of a clique. Luce and Perry (1949) and Luce (1950) are the only well-known sources for such a formalization, and their attempts have not been followed out in the literature. Rather, more recent attempts to satisfactorily identify cliques have employed *ad hoc* clustering or taxonomic procedures which allow the data to suggest natural groupings. Notable among these are the approaches of MacRae (1960), Coleman and MacRae (1960), and Hubbel (1965).

One difficulty in the approach of Luce and Perry (1949) and Luce (1950) was that an adequate computational procedure to locate subsets of the data which satisfied their criteria was lacking. Recent computational literature, such as Bonner (1963) and especially Auguston and Minker (1970), contains algorithms which can be used to identify these subsets in reasonably large bodies of data (whose sizes are on the order of 100 to 1000 individuals).

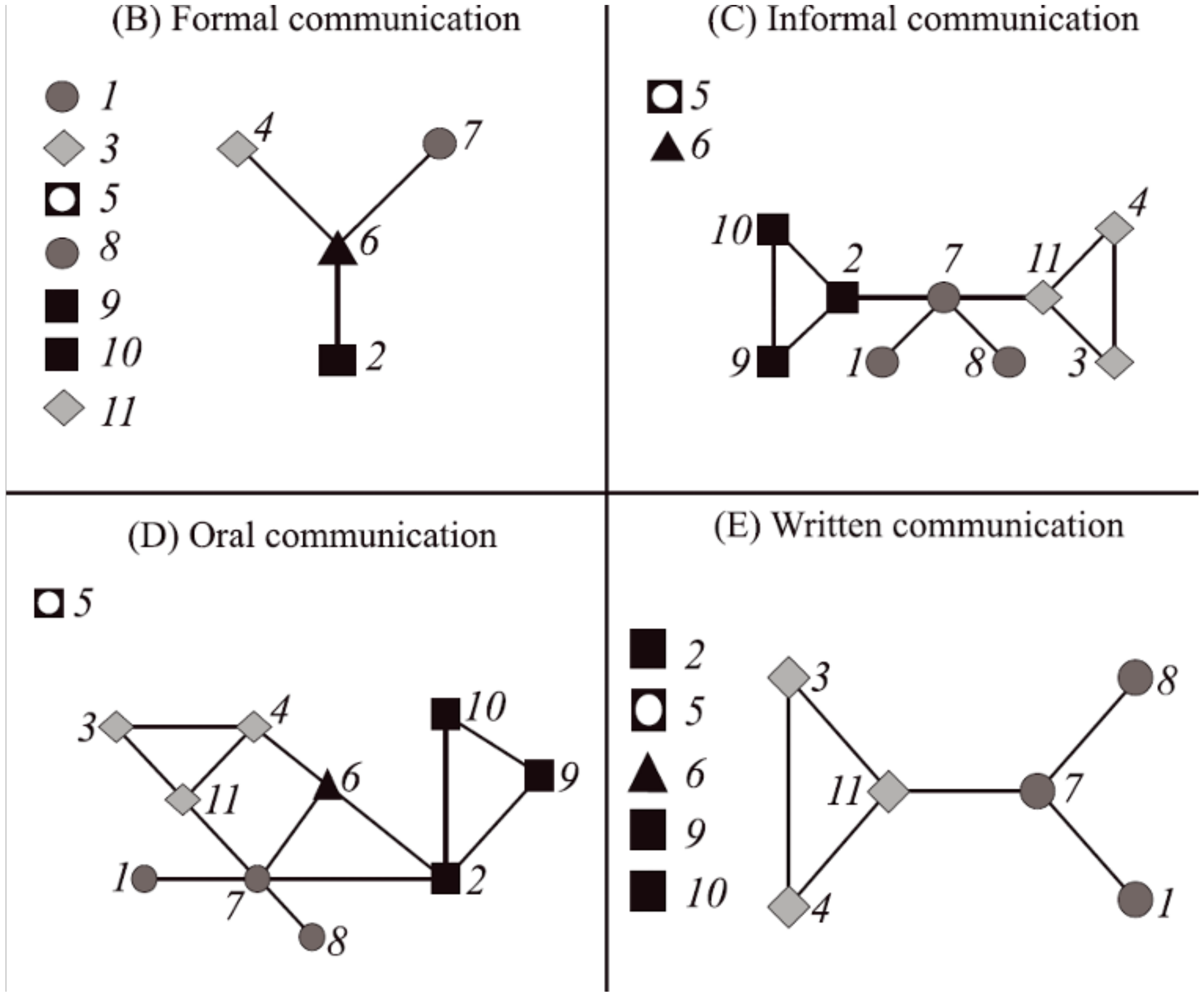
Other difficulties, however, remain. The definition in Luce and Perry (1949), in which a clique is defined as a maximal complete subgraph, is too stringent for most purposes. The concept of n-clique, as presented in Luce (1950), may provide a suitable basis for a formalization but its properties must be explored further before any judgment can be made.

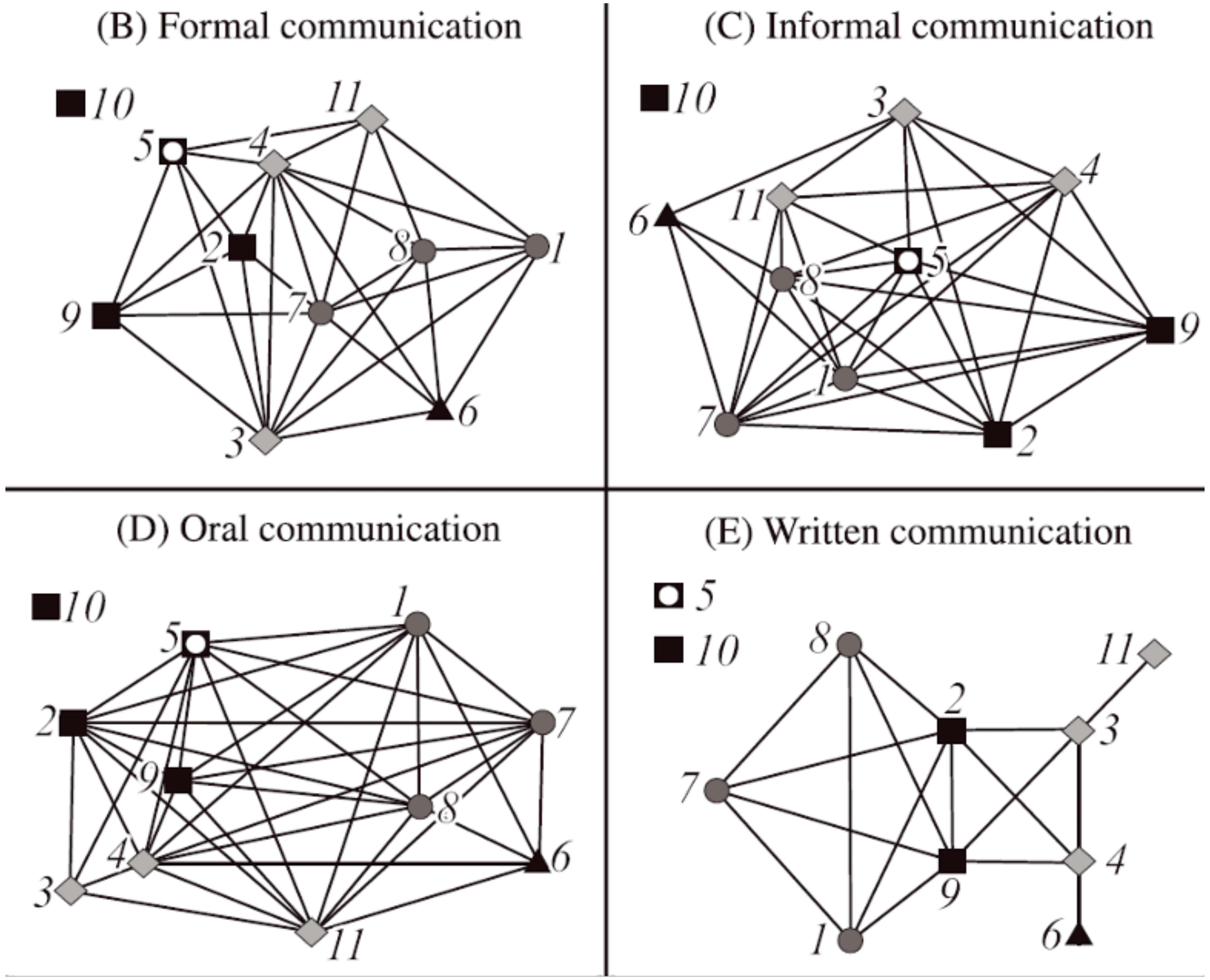
The intent of this paper is to provide a suitable definition of a sociometric clique.

*A version of this paper was read at the 1972 meetings of the American Sociological Association in New Orleans. This paper was prepared under National Science Foundation Grant GS 3231 to Professor Charles Kadushin of Teachers College, Columbia University. I am grateful for the comments of Professor Kadushin, as well as those of Dr. Kenneth Land of the Russell Sage Foundation and Professor Seymour Spilerman of the University of Wisconsin to a draft of this paper. I particularly want to thank Dr. Tom Louis of Columbia University for his help in formulating the applications of probability theory in this paper.

- Luce, R.D., 1950. Connectivity and generalized cliques in sociometric group structure. Psychometrika, 15(2), pp.169-190.
- Alba, R.D., 1973. A graph–theoretic definition of a sociometric clique. Journal of Mathematical Sociology, 3(1), pp.113-126.

APPLICATION EXAMPLE





Espinosa-Curiel, I.E., Rodríguez-Jacobo, J., Vázquez-Alfaro, E., Fernández-Zepeda, J.A. and Fajardo-Delgado, D., 2018. Analysis of the changes in communication and social interactions during the transformation of a traditional team into an agile team. Journal of Software: Evolution and Process, 30(9), p.e1946.

CLIQUE RELAXATIONS

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Clique problem (maximum, maximal, enumeration…)

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INTRODUCTION

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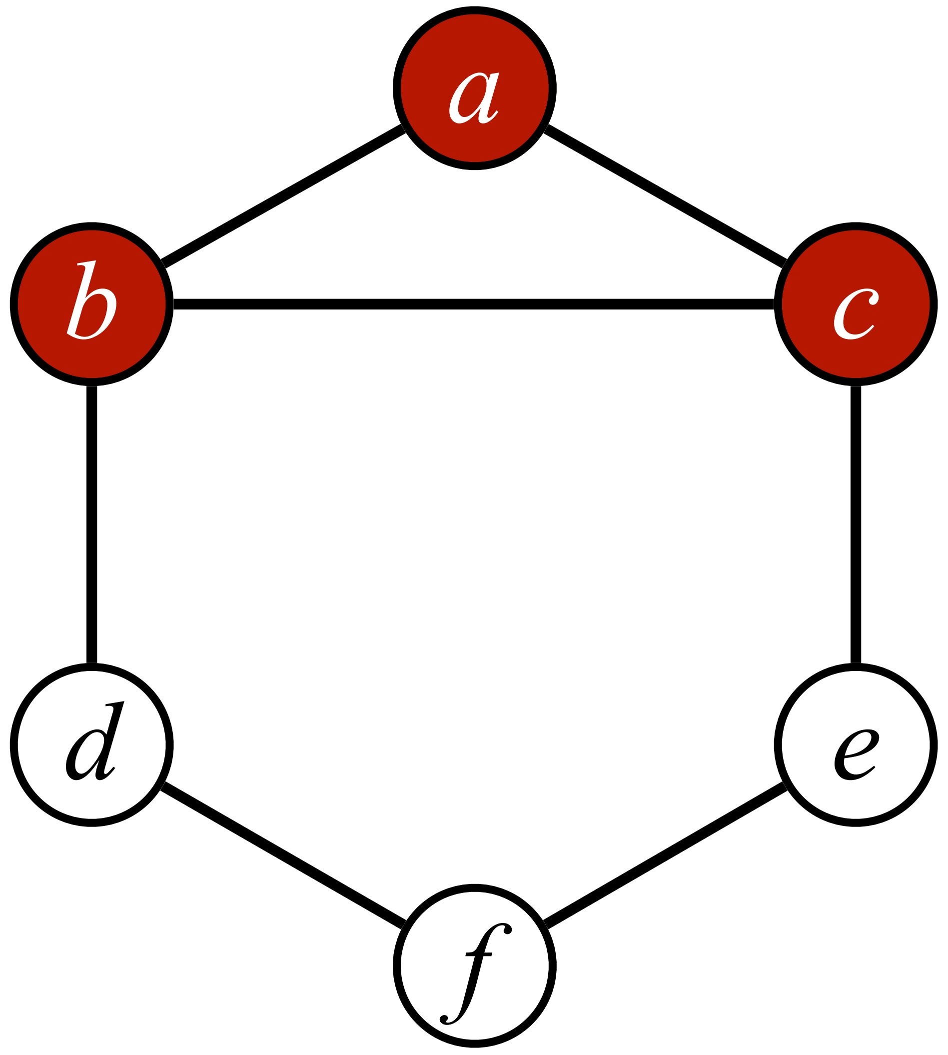
s-CLIQUE

Given a graph $G = (V_G, E_G)$, an ***s*-clique** is a subset of vertices $S \subseteq V_G$ such that:

- The distance between each pair of distinct vertices $u, v \in S$ is at most s edges long.
    - $dist(u, v) \leq s$

Clique as an 1-clique



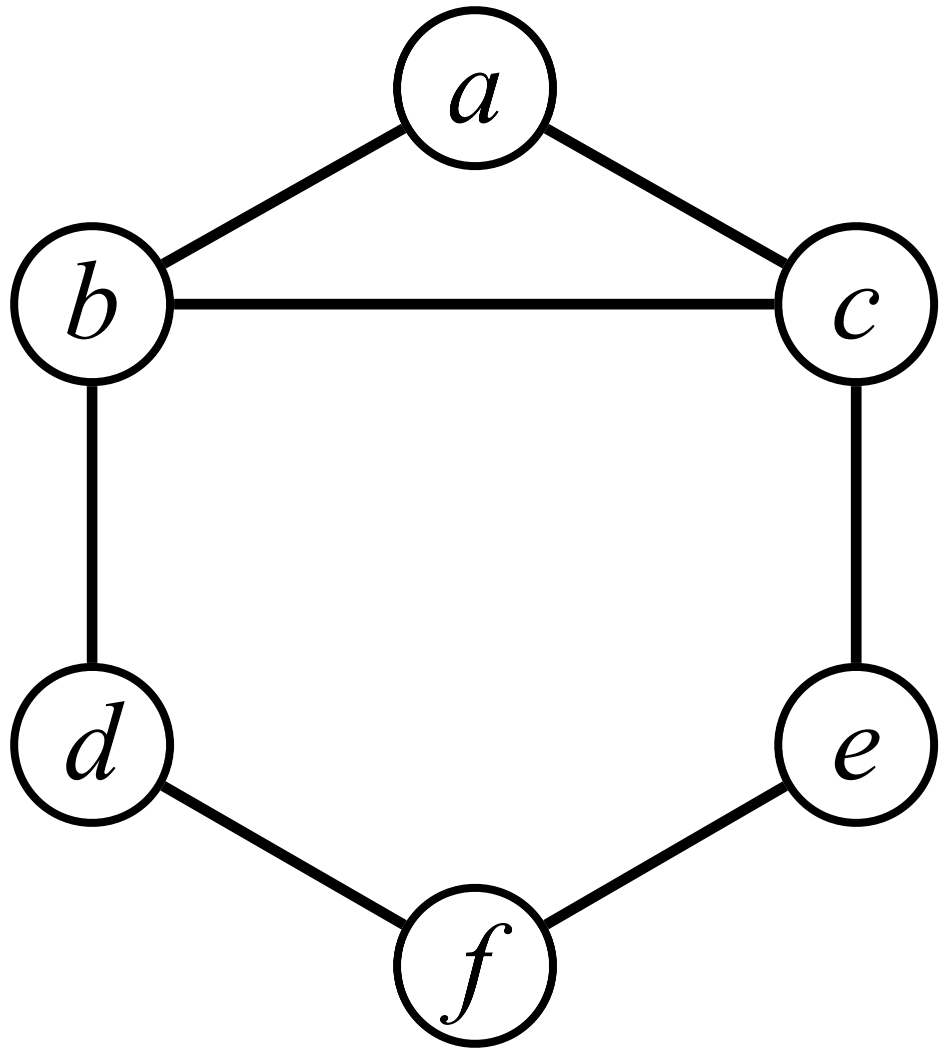
$dist(u, v) \leq 1$

● $\in$ set

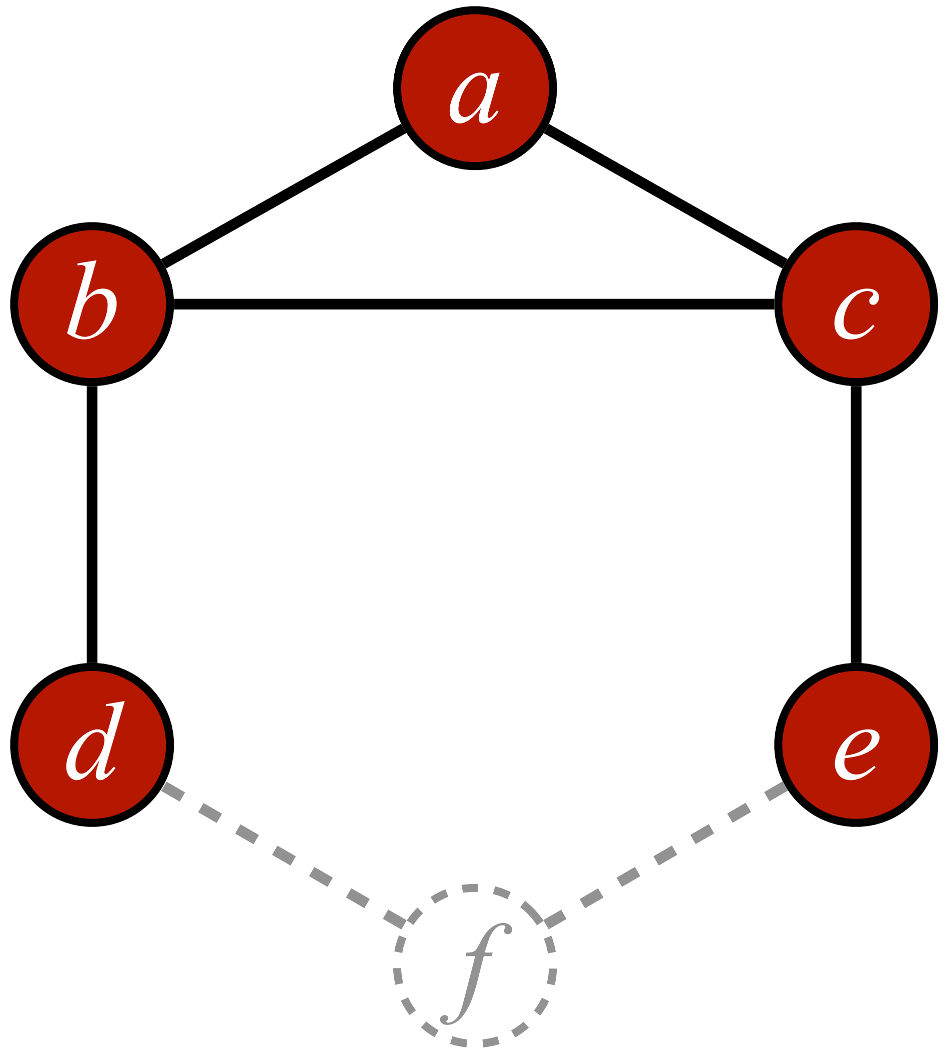
Luce, R.D., 1950. Connectivity and generalized cliques in sociometric group structure. Psychometrika, 15(2), pp.169-190.

2-CLIQUE

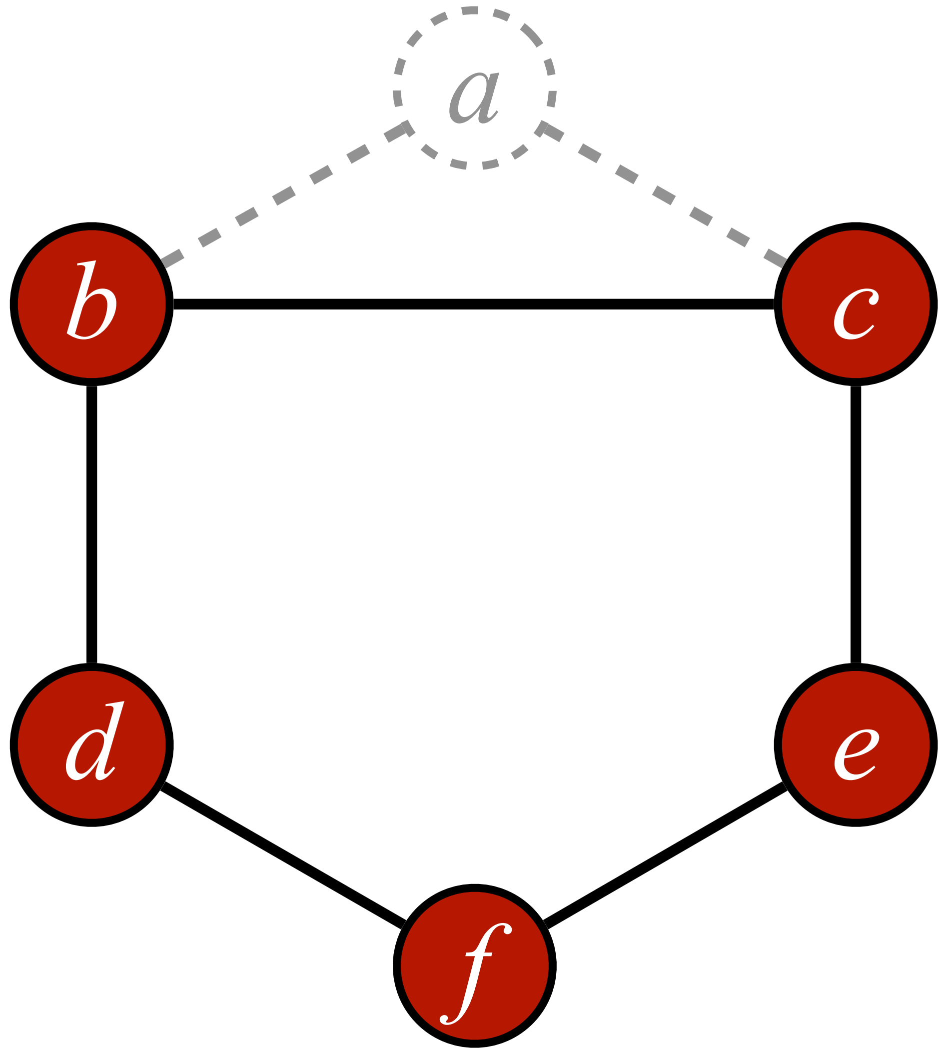
Input graph:
$G = (V_G, E_G)$

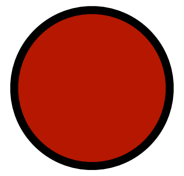


$S^1 = \{a, b, c, d, e\}$



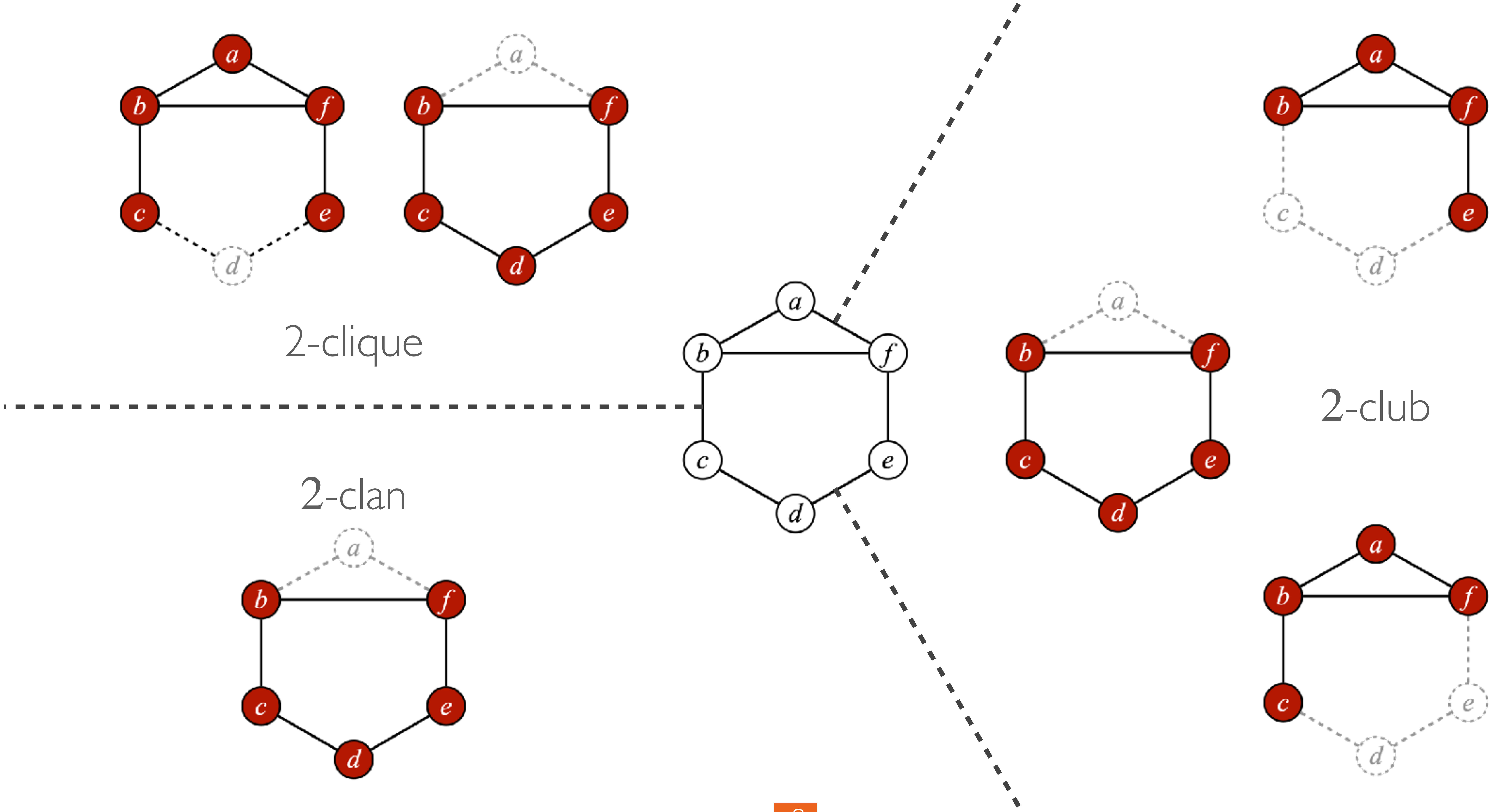
$S^2\{b, c, d, e, f\}$



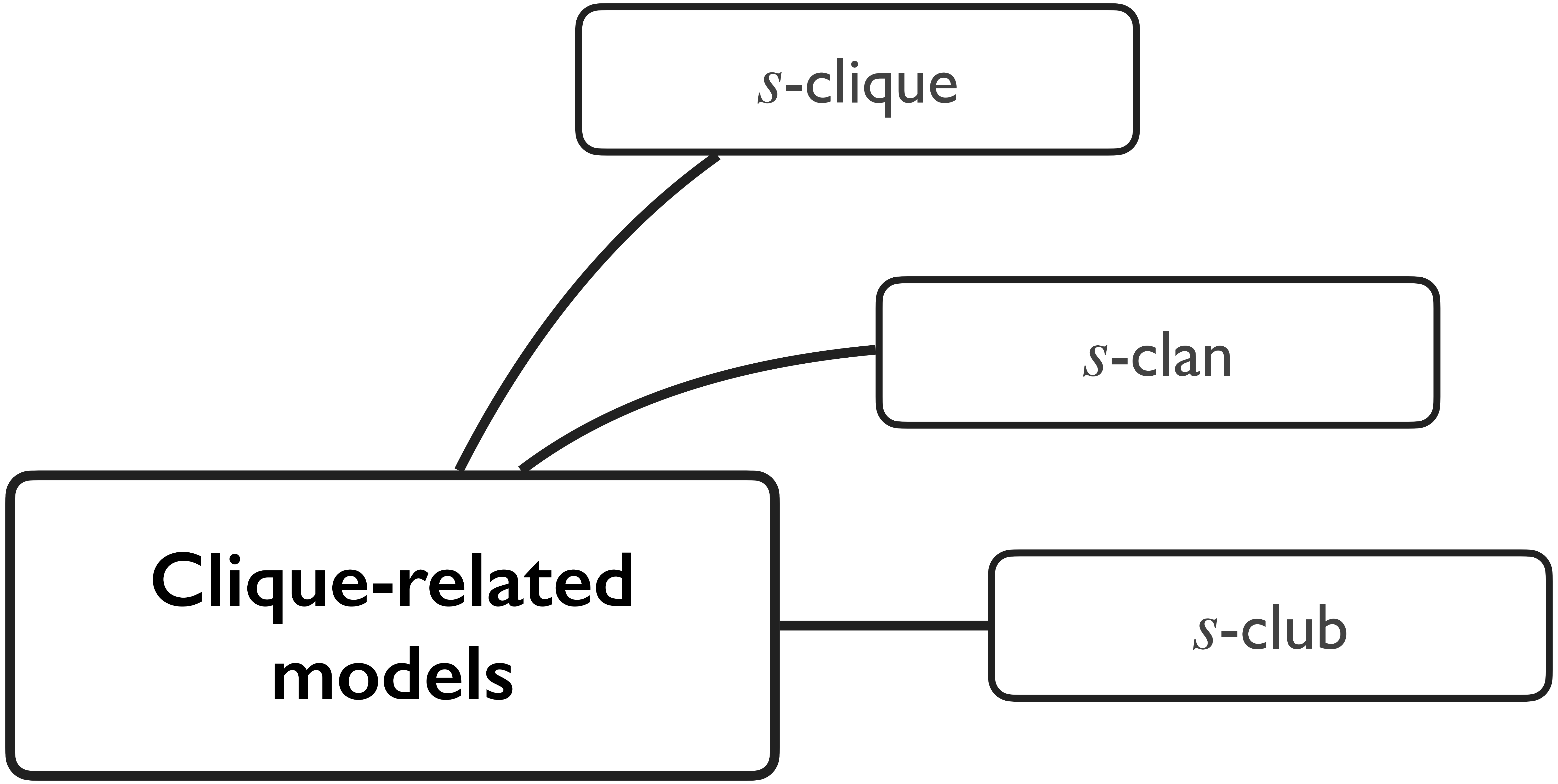
 $\in$ set

$dist(d, e)$ uses edges
$\{(d, f), (f, e)\}$

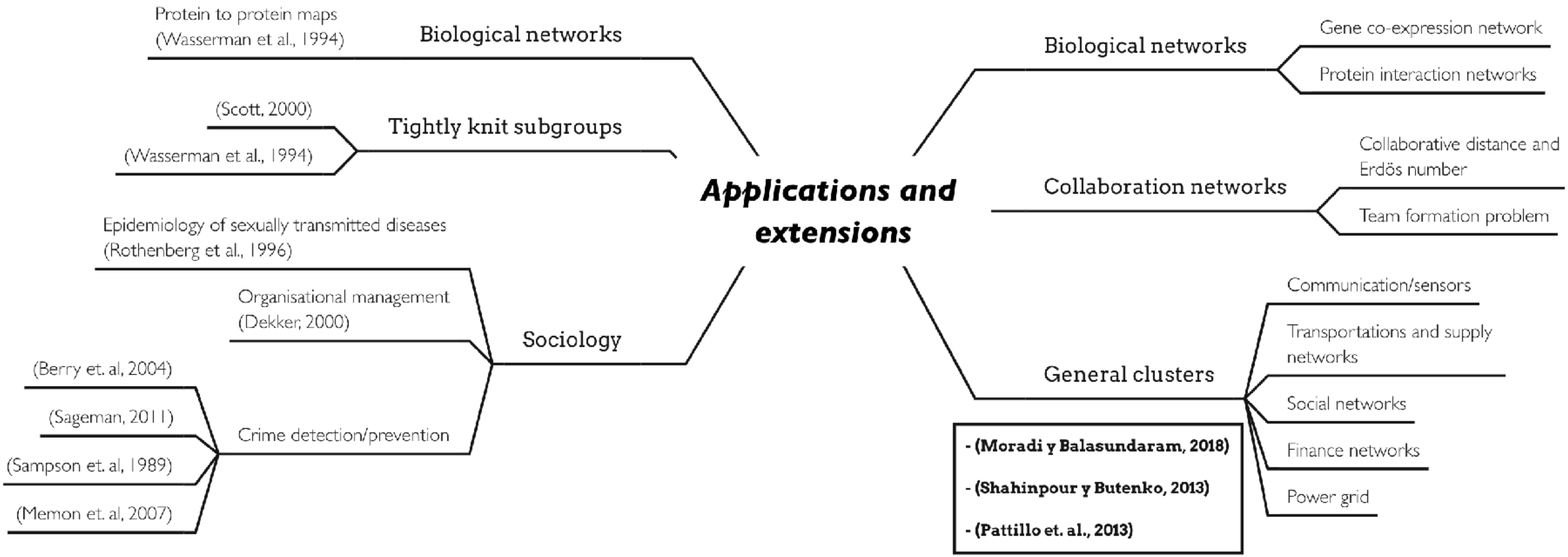
CLIQUES, CLUBS, CLANS

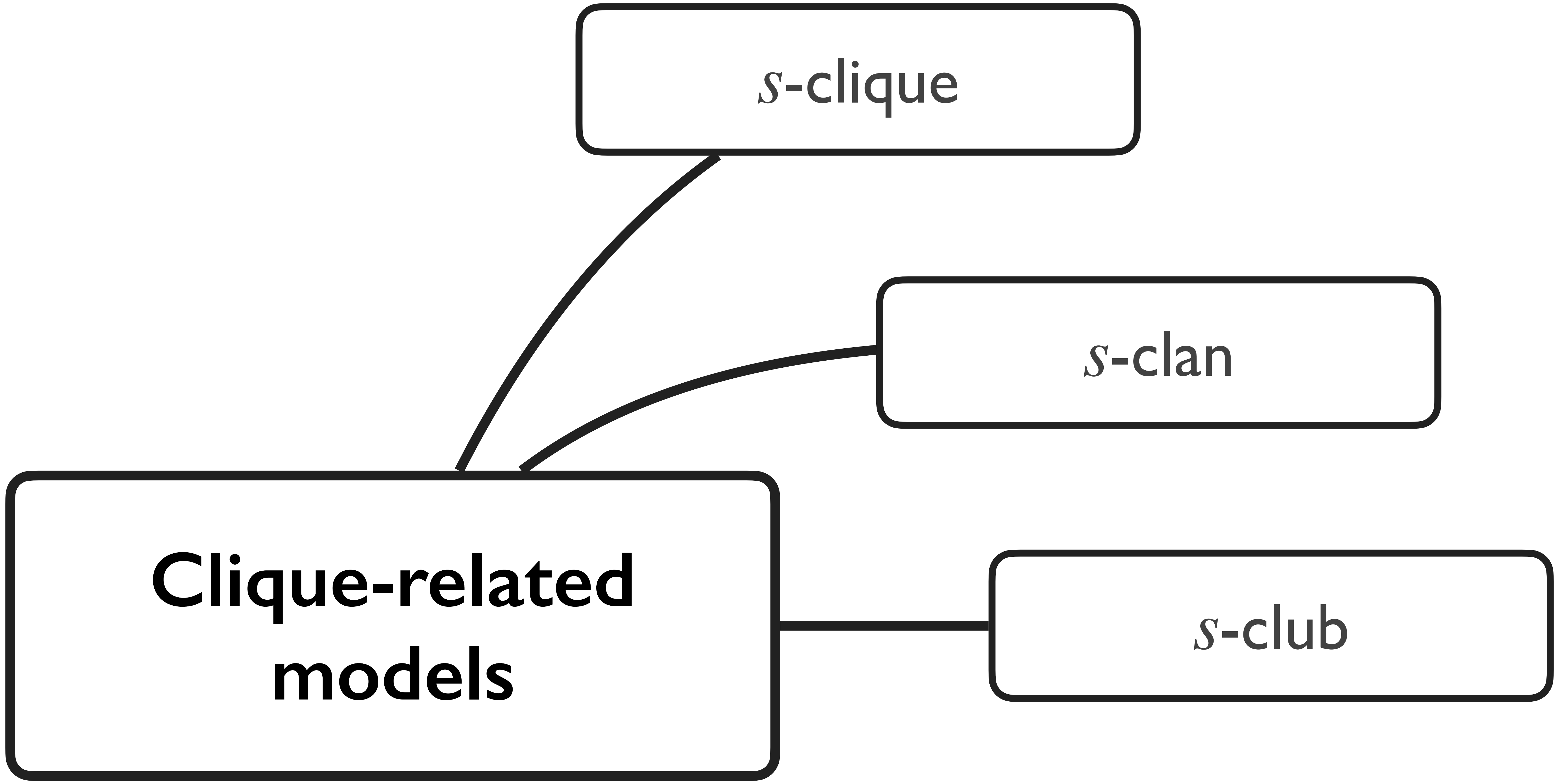


Clique problem (maximum, maximal, enumeration…)

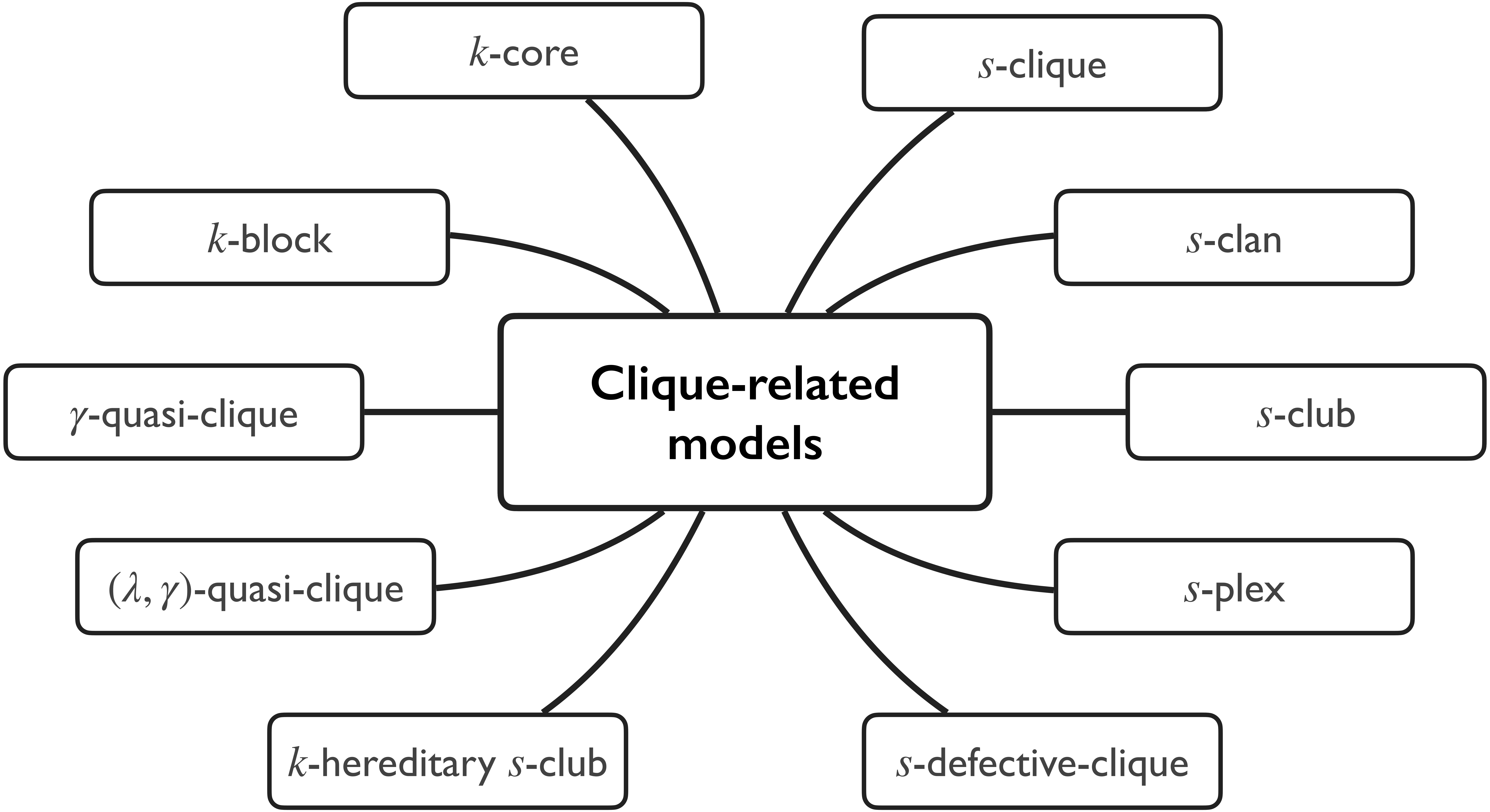

s-clique
s-clan
Clique-related models
s-club

APPLICATIONS AND EXTENSIONS

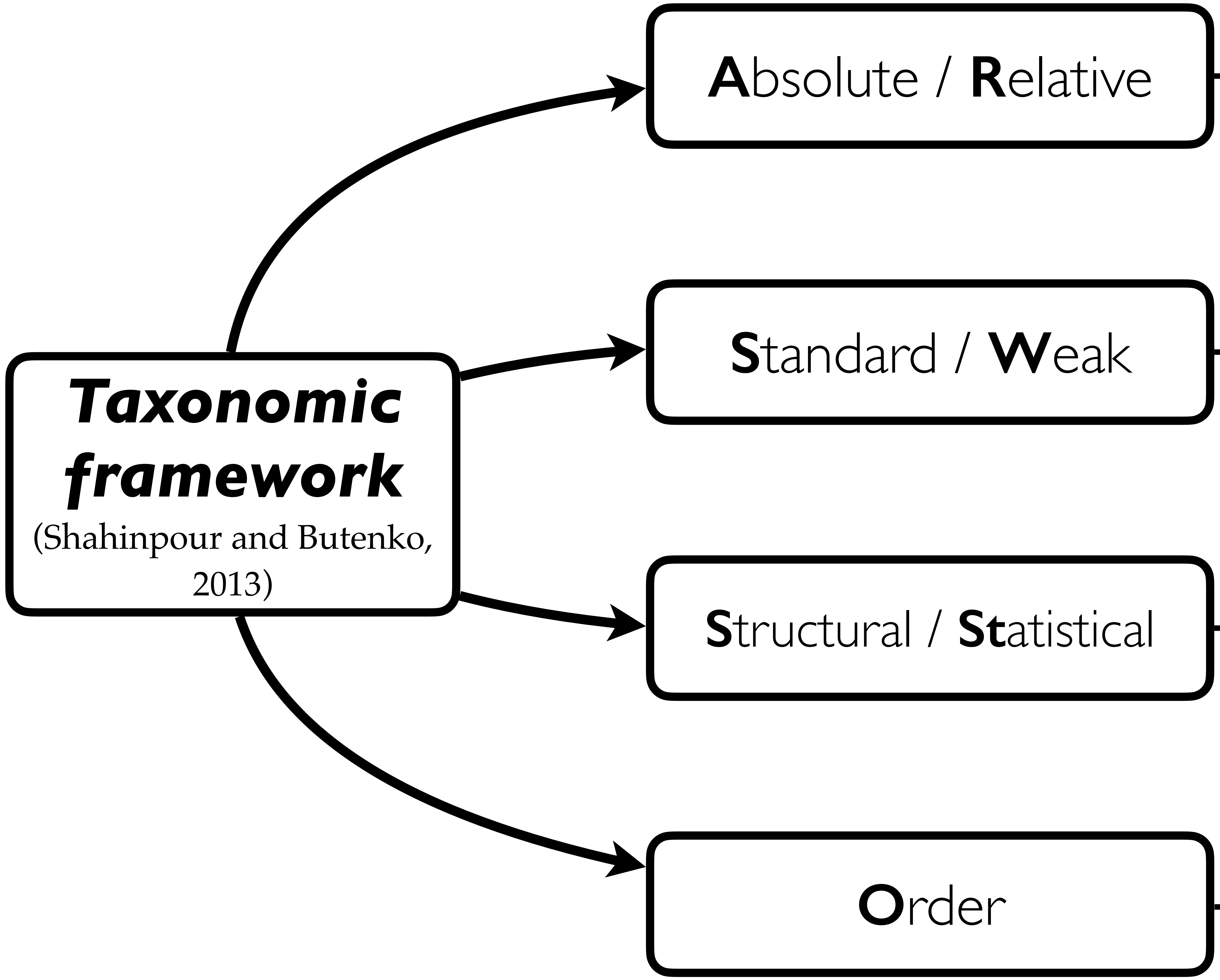




Clique-related models
s-clique
s-clan
s-club


k-core
s-clique
k-block
s-clan
Clique-related models
γ-quasi-clique
s-club
(λ, γ)-quasi-clique
s-plex
k-hereditary s-club
s-defective-clique

Framework



A. Uses absolute parameter values (s or k)

R. Replaces s or k by $\gamma|S|$, $0 \leq \gamma \leq 1$.

S. A property Π holds in $G[S]$

W. A property Π holds in G

S. All members of the set satisfy the property Π.

St. The members of the set satisfy, on average, the property Π.

0. It is the ***clique problem***

1er. It relaxes one of the above characteristics

2do. It relaxes two of the above characteristics

… …

Pattillo, J., Youssef, N. and Butenko, S., 2013. On clique relaxation models in network analysis. European Journal of Operational Research, 226(1), pp.9-18.

The maximum s-clique problem on trees (this talk)

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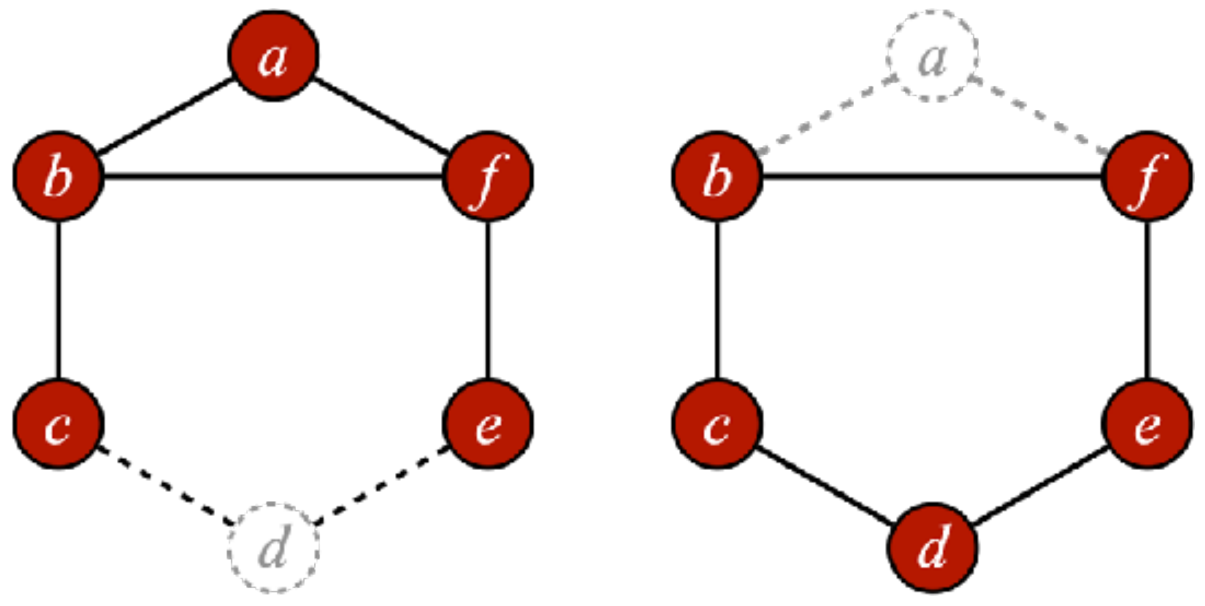
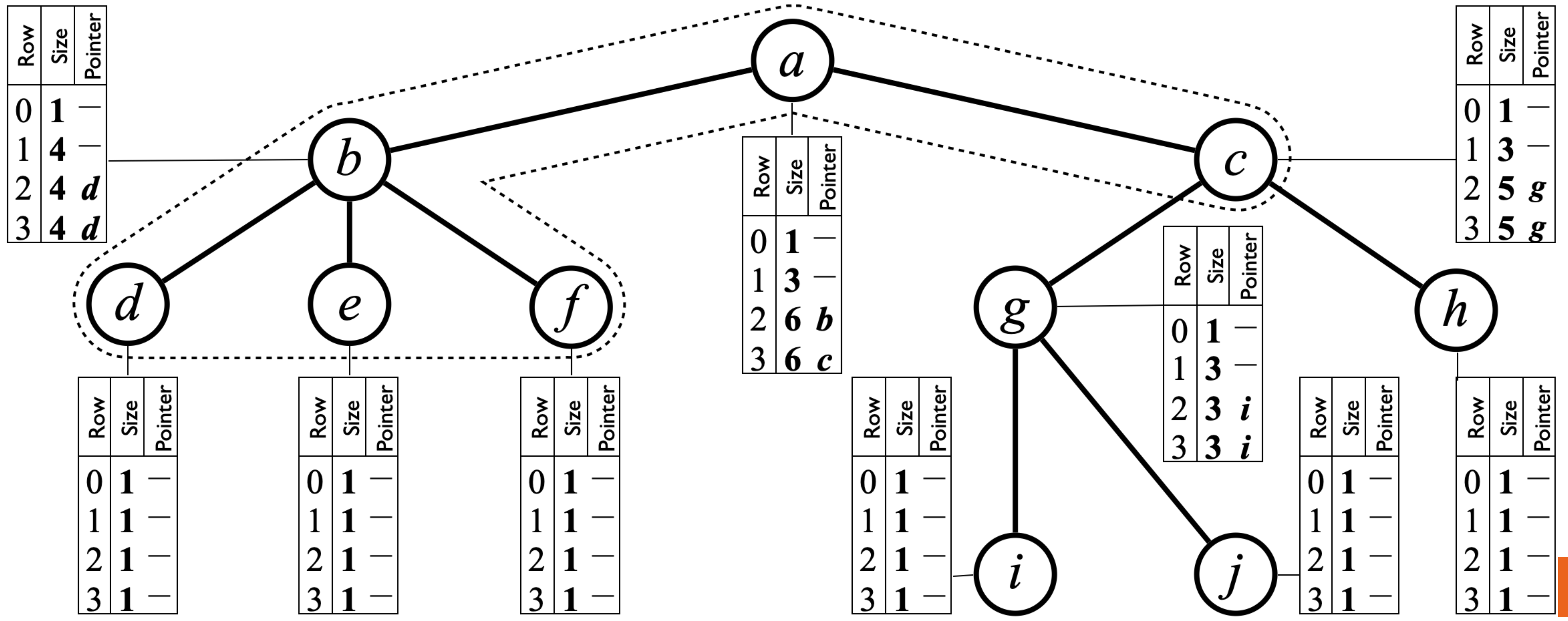
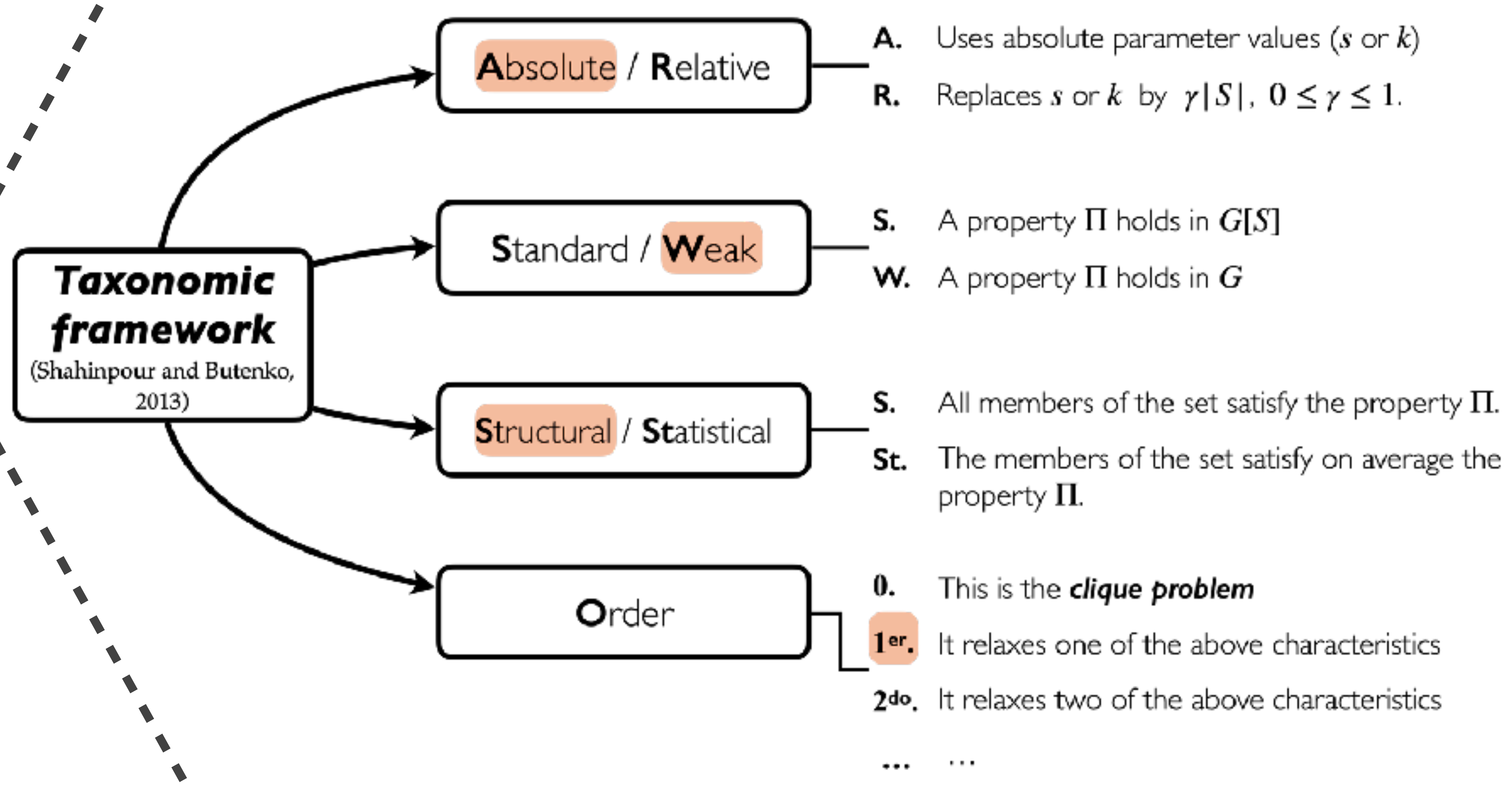
s-clique problem

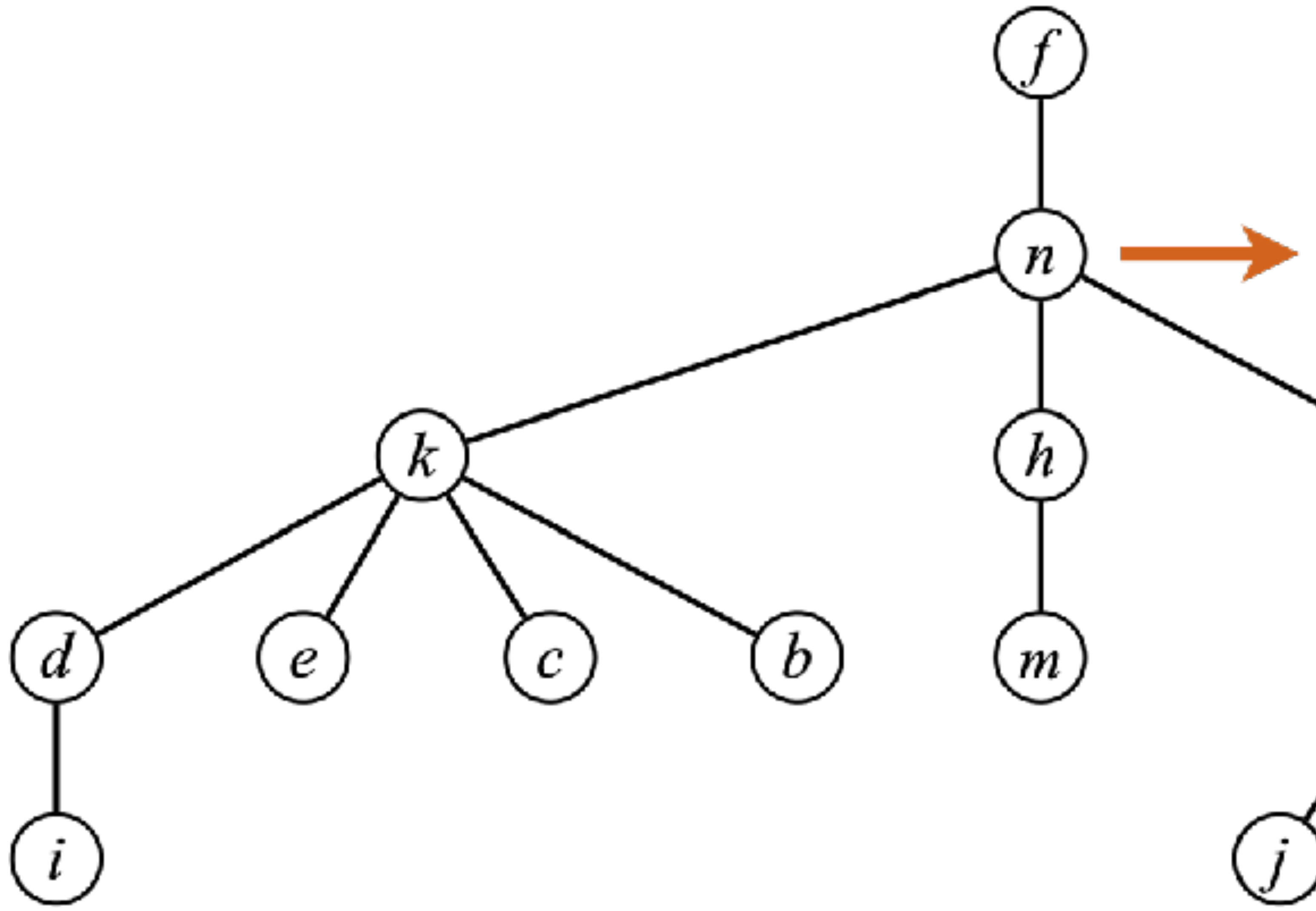


Dynamic programming





STRATEGY



Row	*Size*	*Pointer*
0		
1		
2		
3		
4		
5		

Let $T = (V_T, E_T)$ be a tree graph, $v \in V_T$ and s an integer.

For this example, consider $s = 5$

Each vertex v has an associated table with the following:

- $s + 1$ rows
- 2 columns: ***Size*** and ***Pointer***

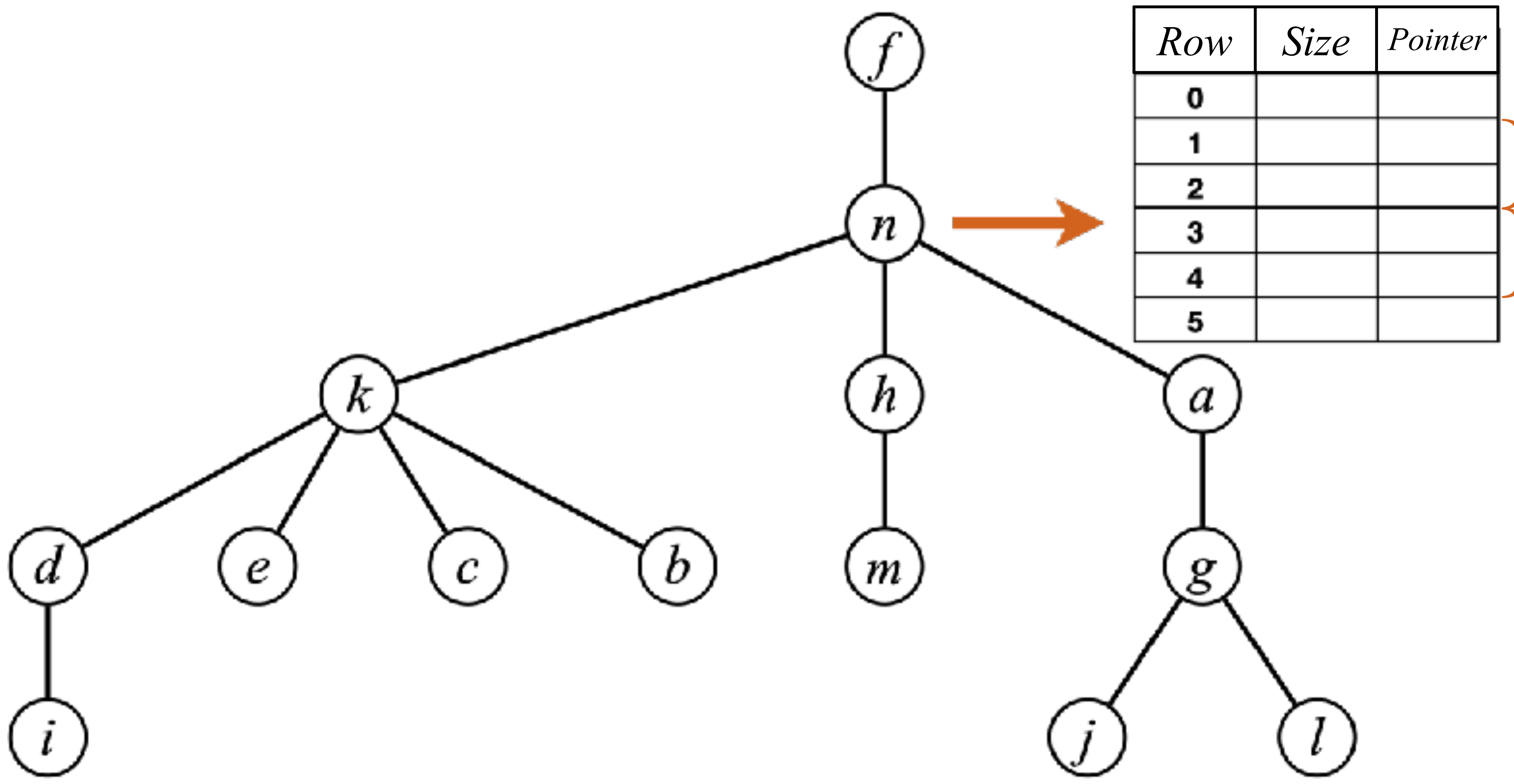
If v is a leaf vertex, trivial case:

$$t\,.\,table[row]\,.\,size \leftarrow 1, \forall row \in [s]$$

Otherwise, there are **four cases**:

- $row = 0$
- $1 \leq row \leq \left\lfloor \frac{s}{2} \right\rfloor$
- $\left\lfloor \frac{s}{2} \right\rfloor < row \leq s - 1$
- $row = s$

STRATEGY



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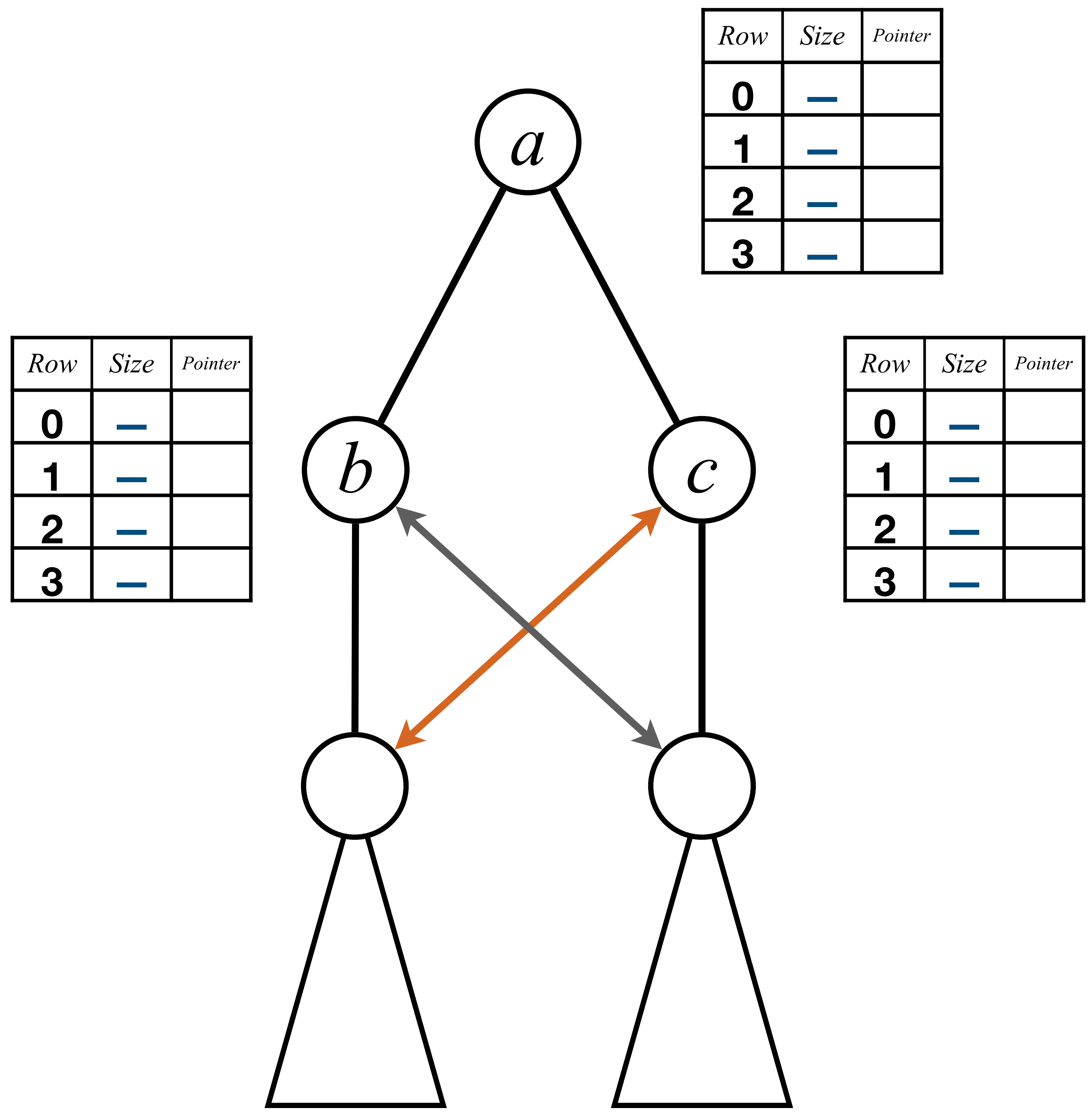
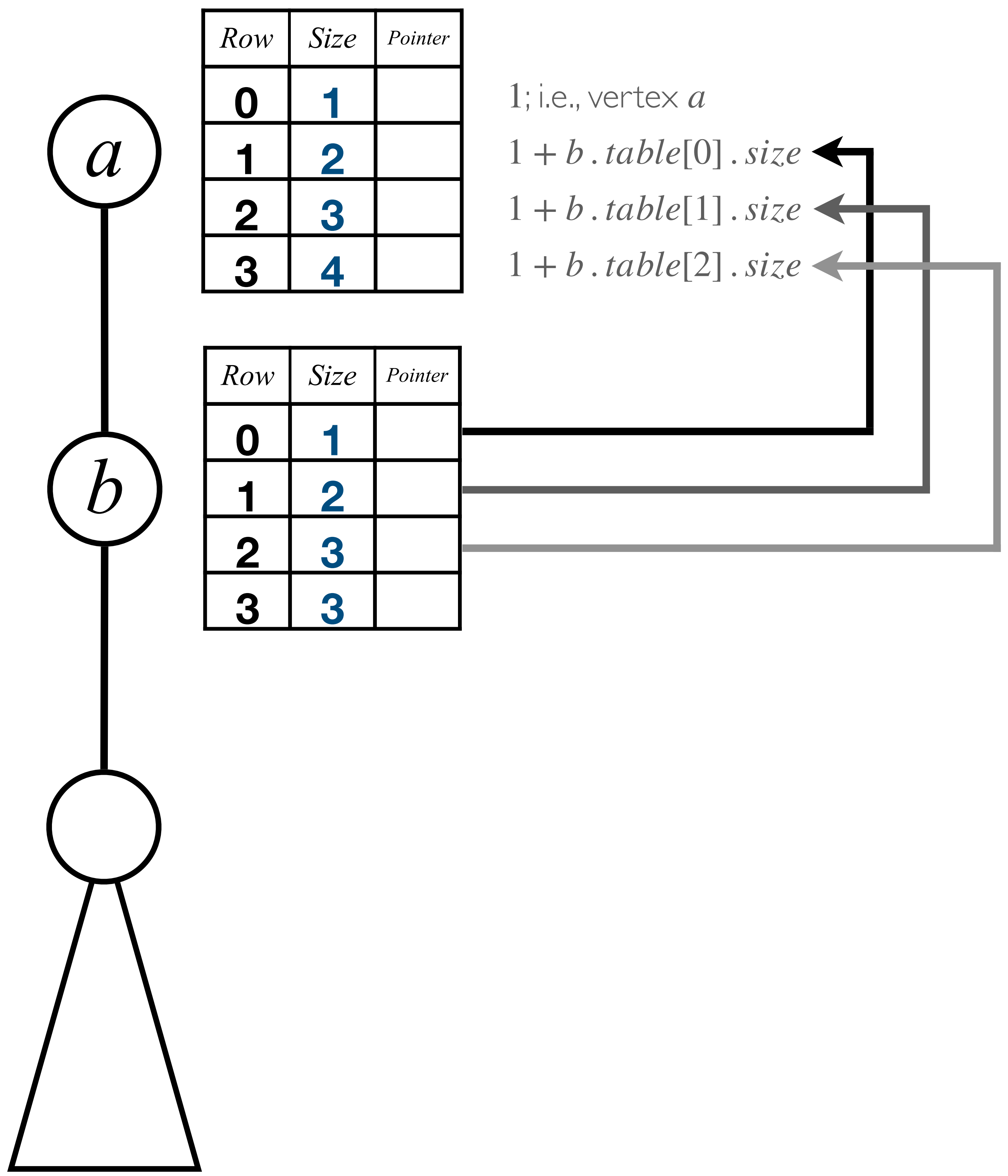


Case	Action
1) $row = 0$	$a\,.\,table[0]\,.\,size \leftarrow 1$
2) $1 \le row \le \left\lfloor \frac{s}{2} \right\rfloor$	$a\,.\,table[row]\,.\,size \leftarrow 1 + \sum_{w \in C_a} w\,.\,table[row-1]\,.\,size$
3) $\left\lfloor \frac{s}{2} \right\rfloor < row \le s-1$	$total \leftarrow \sum_{\forall w \in C_a} w\,.\,table[s-row-1]$ $a\,.\,table[row]\,.\,size \leftarrow 1 + \max_{\forall w \in C_a} \left\{ total - \left(w\,.\,table[s-row-1]\,.\,size + w\,.\,table[row-1]\,.\,size\right) \right.$
4) $row = s$	$a\,.\,table[s]\,.\,size \leftarrow 1 + \max_{\forall w \in C_a} \{w\,.\,table[s-1]\,.\,size$

$a\,.\,table[row]\,.\,size \leftarrow \max\left(a\,.\,table[row]\,.\,size, a\,.\,table[row-1]\,.\,size\right)$, for $1 \le row \le s$

Update $a\,.\,table[row]\,.\,pointer \leftarrow a\,.\,table[row-1]\,.\,pointer$, accordingly

STRATEGY



Case	Action
1) $row = 0$	$a.table[0].size \leftarrow 1$
2) $1 \leq row \leq \left\lfloor \frac{s}{2} \right\rfloor$	$a.table[row].size \leftarrow 1 + \sum_{w \in C_a} w.table[row-1].size$
3) $\left\lfloor \frac{s}{2} \right\rfloor < row \leq s-1$	$total \leftarrow \sum_{\forall w \in C_a} w.table[s-row-1]$ $a.table[row].size \leftarrow 1 + \max_{\forall w \in C_a} \left\{ total - \left(w.table[s-row-1].size + w.table[row-1].size \right) \right.$
4) $row = s$	$a.table[s].size \leftarrow 1 + \max_{\forall w \in C_a} \{ w.table[s-1].size$

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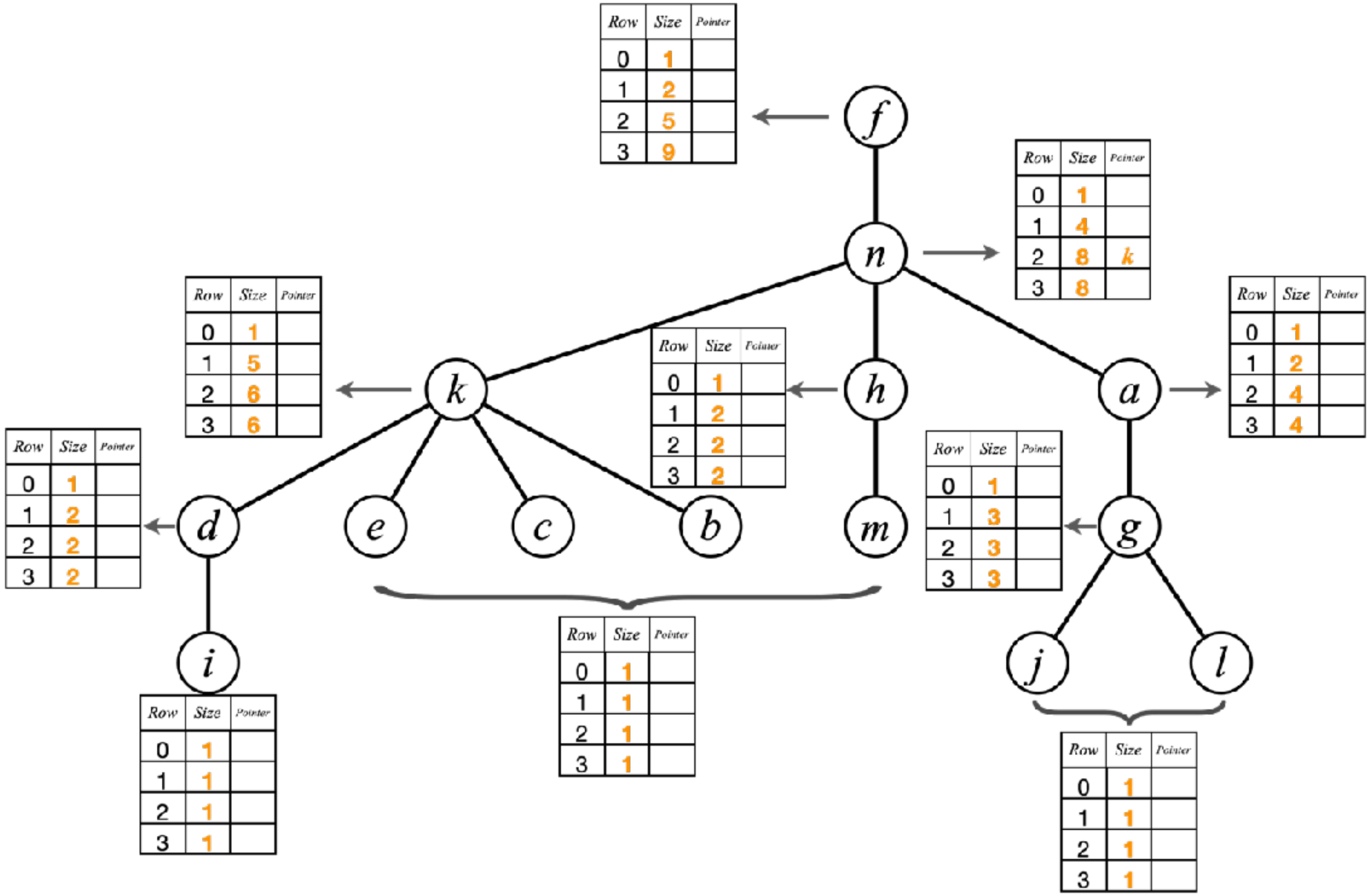
f
n
k
h
a
d
e
c
b
m
g
i
j
l
Row Size Pointer
0 1
1 2
2 5
3 9
Row Size Pointer
0 1
1 4
2 8 k
3 8
Row Size Pointer
0 1
1 5
2 6
3 6
Row Size Pointer
0 1
1 2
2 2
3 2
Row Size Pointer
0 1
1 2
2 4
3 4
Row Size Pointer
0 1
1 2
2 2
3 2
Row Size Pointer
0 1
1 3
2 3
3 3
Row Size Pointer
0 1
1 1
2 1
3 1
Row Size Pointer
0 1
1 1
2 1
3 1
Row Size Pointer
0 1
1 1
2 1
3 1

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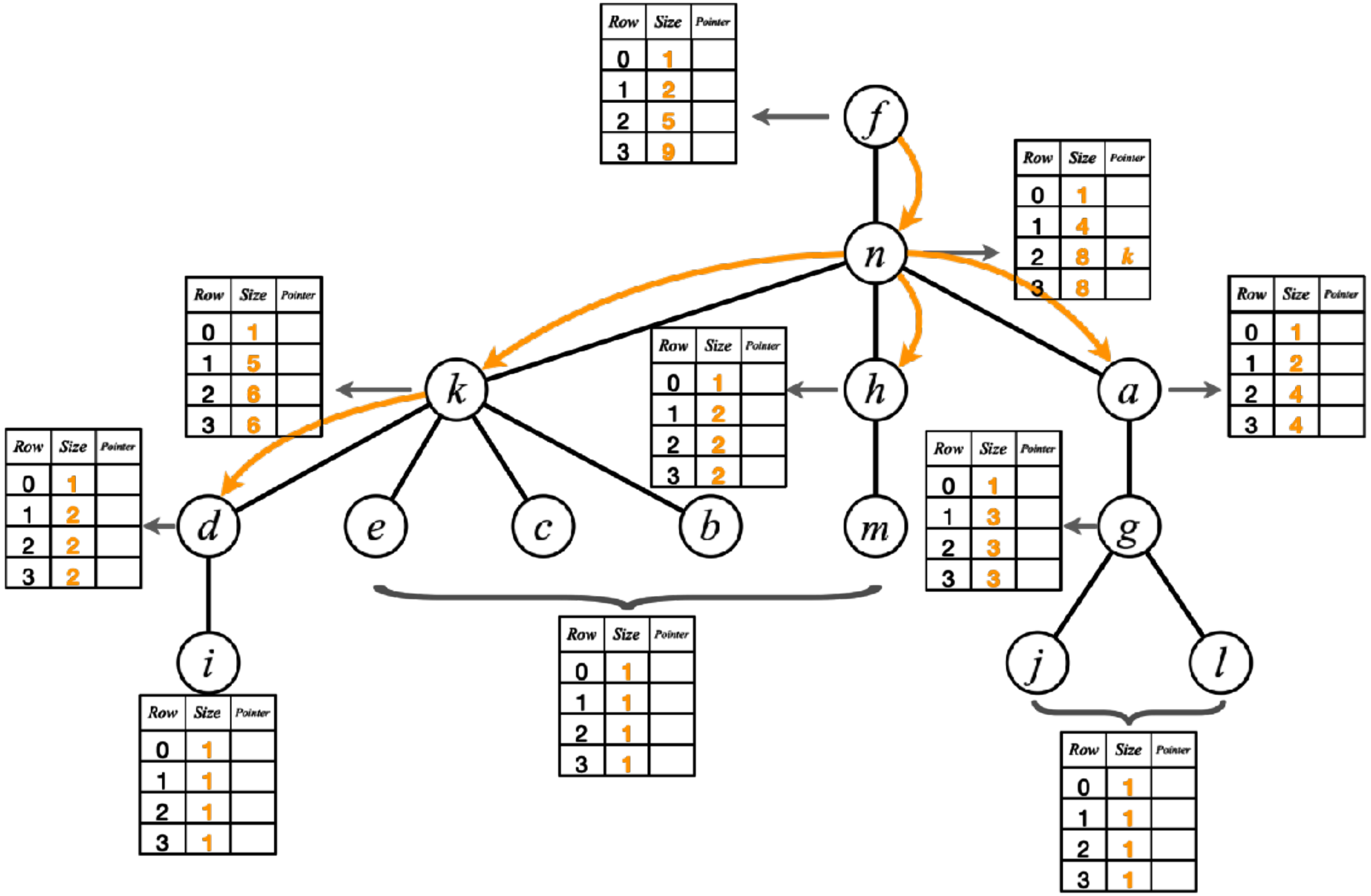
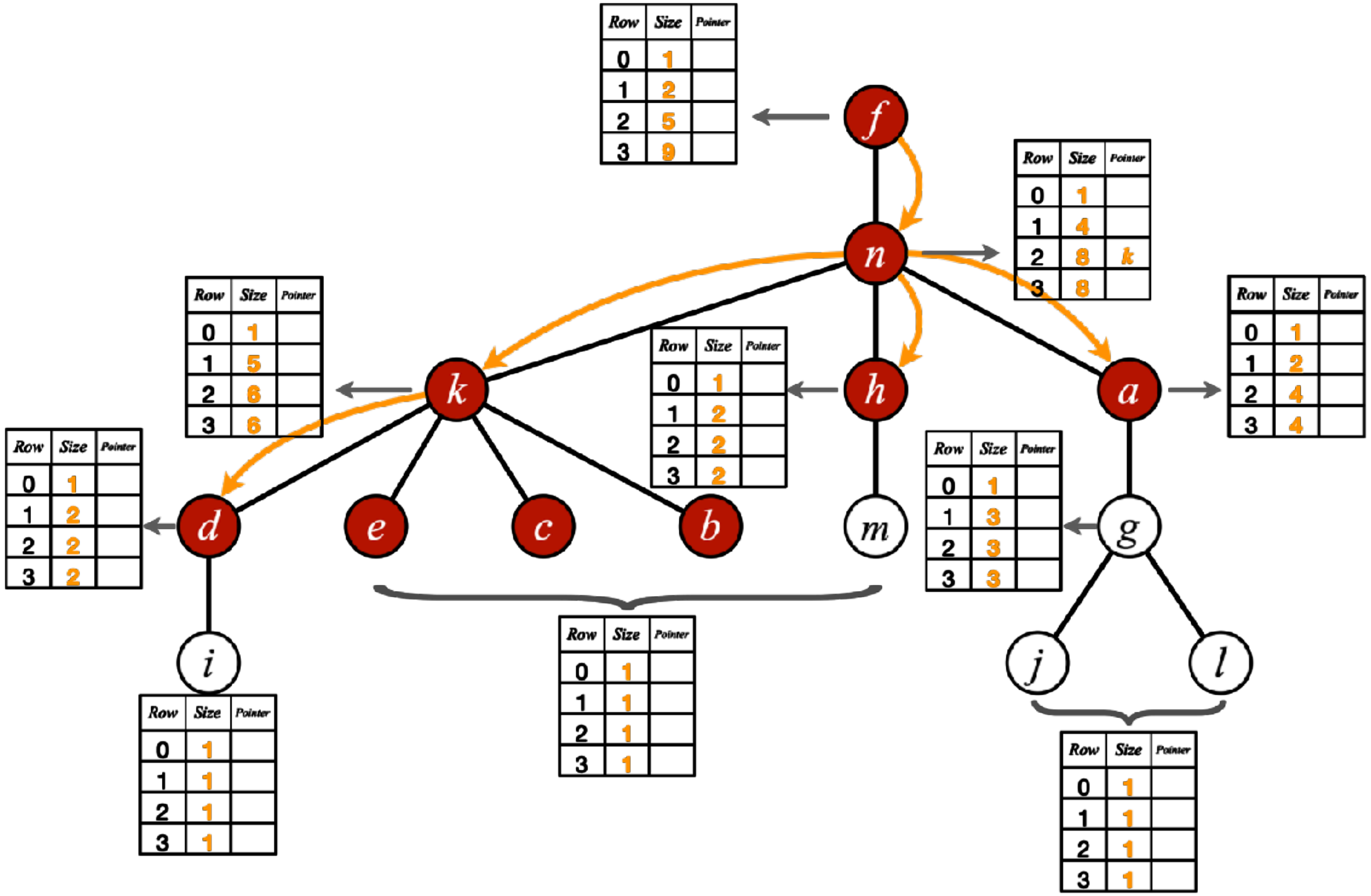
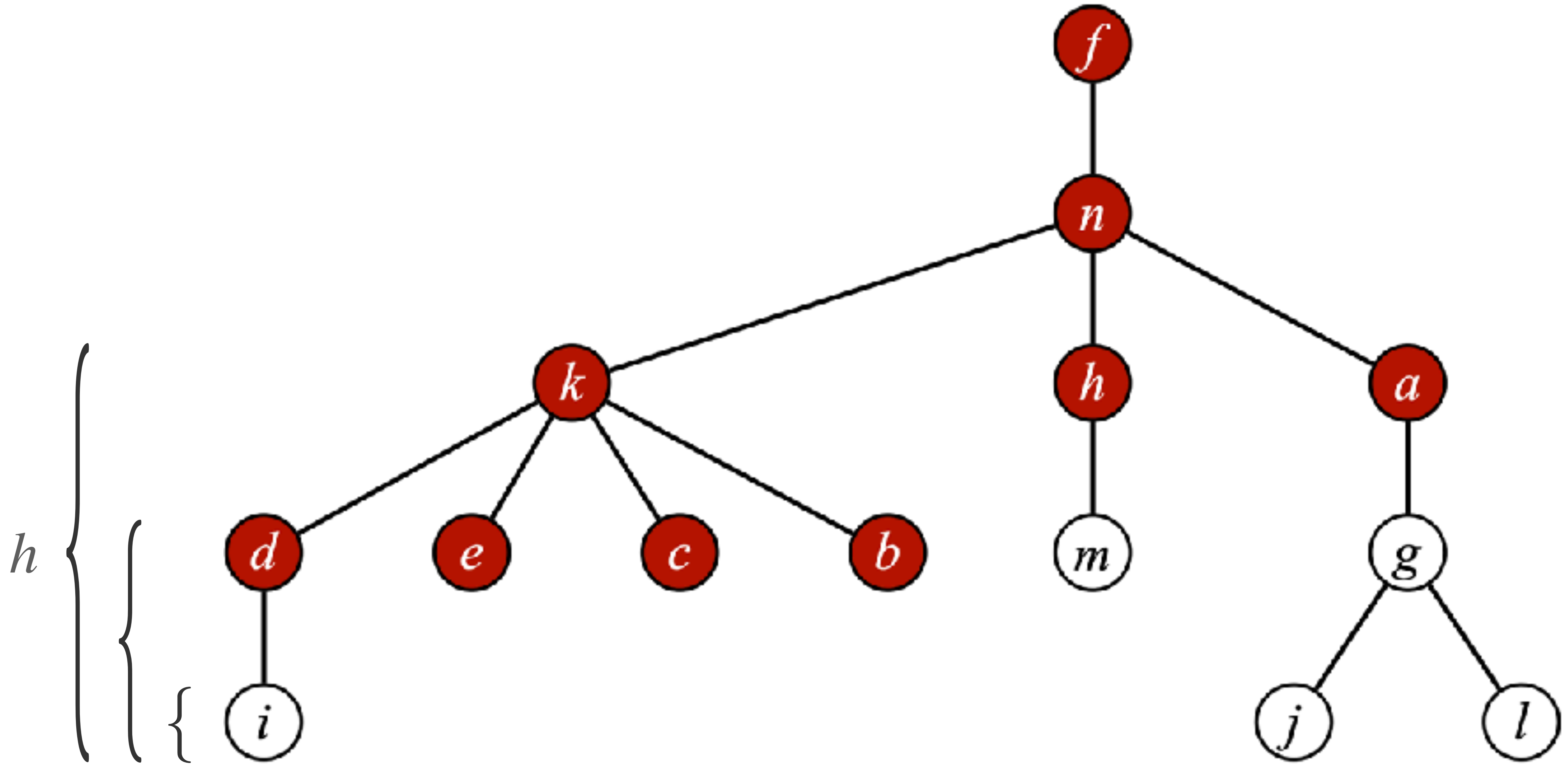

f
n
k
h
a
d
e
c
b
m
g
i
j
l
Row Size Pointer
0 1
1 2
2 5
3 9
Row Size Pointer
0 1
1 4
2 8 k
3 8
Row Size Pointer
0 1
1 5
2 6
3 6
Row Size Pointer
0 1
1 2
2 2
3 2
Row Size Pointer
0 1
1 2
2 4
3 4
Row Size Pointer
0 1
1 2
2 2
3 2
Row Size Pointer
0 1
1 3
2 3
3 3
Row Size Pointer
0 1
1 1
2 1
3 1
Row Size Pointer
0 1
1 1
2 1
3 1
Row Size Pointer
0 1
1 1
2 1
3 1

Strategy



f
n
k
h
a
d
e
c
b
m
g
i
j
l
Row Size Pointer
0 1
1 2
2 5
3 9
Row Size Pointer
0 1
1 4
2 8 k
3 8

CORRECTNESS AND ANALYSIS



- Filling in the table of each vertex v: → $O\left(s \cdot |C_v|\right)$ u.t.
- Traversals on the tree: → $O(n)$ u.t.
- Total execution time: → $O(s \cdot n)$ u.t.

OTHER METHODOLOGIES

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Methodologies for the s-clique

- **Exact**
  - Bron & Kerbosh, 1973
    - Clique, arbitrary graph
    - $O(3^{\frac{n}{3}})$ u.t.
  - Schäfer, 2009
    - s-club, trees
    - $O(s^2 \cdot n)$ u.t.
  - ***This work***
    - s-clique, trees
    - $O(s \cdot n)$ u.t.
  - Power of a graph
    - s-clique, arbitrary graph
    - Exponential
- **Heuristics**
  - Edachery, et, al., 1999
    - s-clique (clusters), arbitrary graph
    - $O(n^3)$ u.t.
  - Behar & Cohen, 2018
    - ~ Bron y Kerbosh
- **Mathematical formulation**
  - Balasundaram, et, al., 2005*
    - Arbitrary graph
    - CPLEX
    - Small values for s

SCHÄFER'S ALGORITHM & OUR PROPOSAL

ROYAL HOLLOWAY UNIVERSITY OF LONDON

Schäfer's

The maximum s-club problem on trees

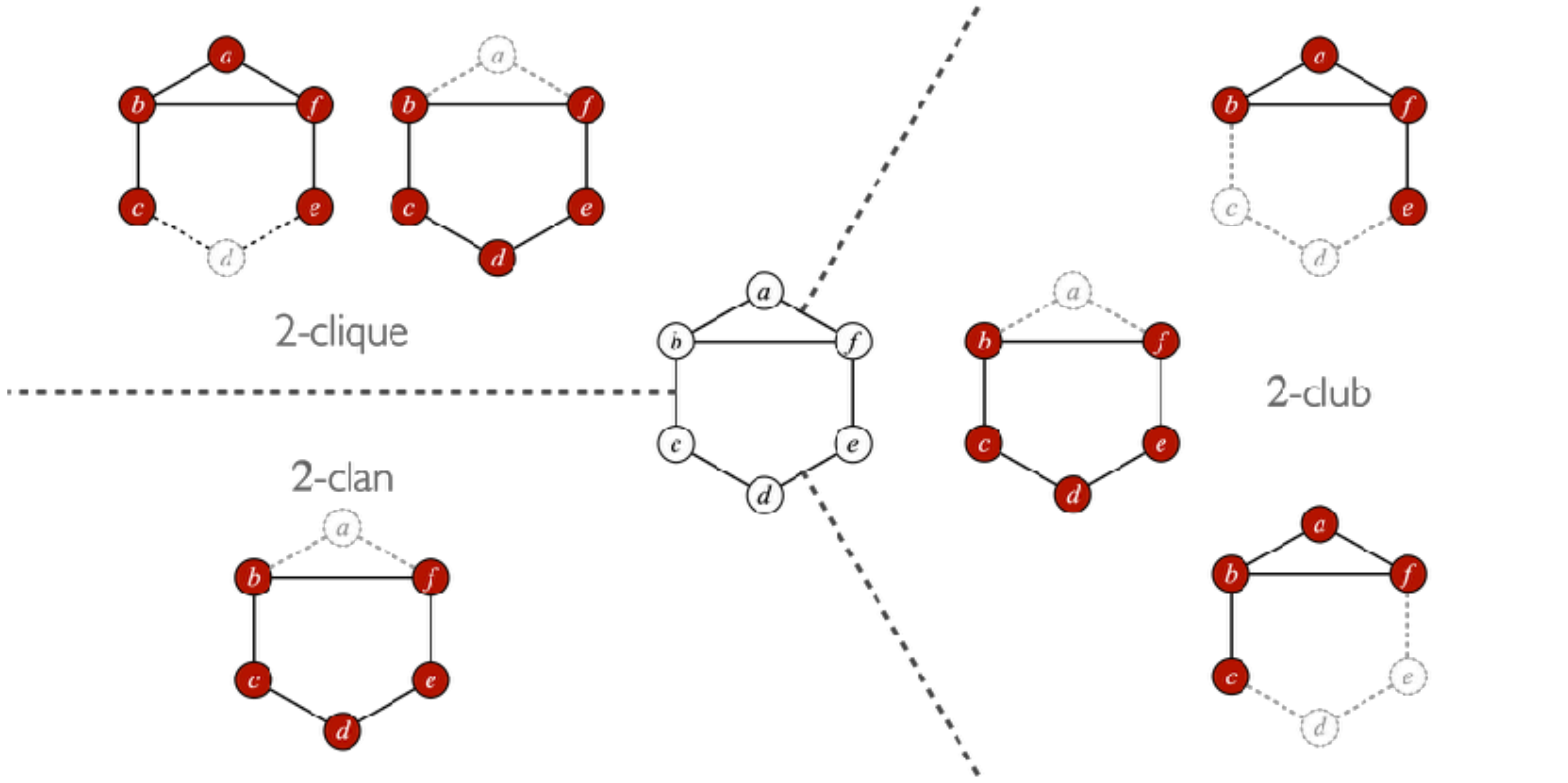
$$T(n) = O\left(s^2 \cdot n\right)$$

Our proposal

The maximum s-clique problem on trees

$$T(n) = O\left(s \cdot n\right)$$

On trees: the s-clan, s-club, and s-clique problems are equivalent



Schäfer, A.: Exact algorithms for s-club finding and related problems. Ph.D. thesis, Friedrich-Schiller-University Jena (2009)

EXPERIMENTS

Data set

1. $|\mathscr{T}_{22\_16}| = 12{,}761$, $|V_T| = 22$, Prof. Brendan McKay.
2. T_D, a doubly logarithmic tree of height 5 ($|V_T| = 119{,}041$)
3. T_L a linear tree ($|V_T| = 10{,}000$)
4. T_B a full balanced binary tree with 2^{16} leaves
5. $|\mathscr{T}_{Ph}| = 5{,}722$, $|V_T| =$ up to 297, PhylomeDB
6. $T_{\eta e6}$ for $\eta \in \{0.3, 0.5, 0.75, 1\}$ and $e6 = 10^6$ nodes

$$s \in \left\{ \begin{array}{l} 4, 5, 10, 15, 20, \\ 25, 30, 100, 500, \\ 1\,000, 10\,000 \end{array} \right\}$$

where appropriate

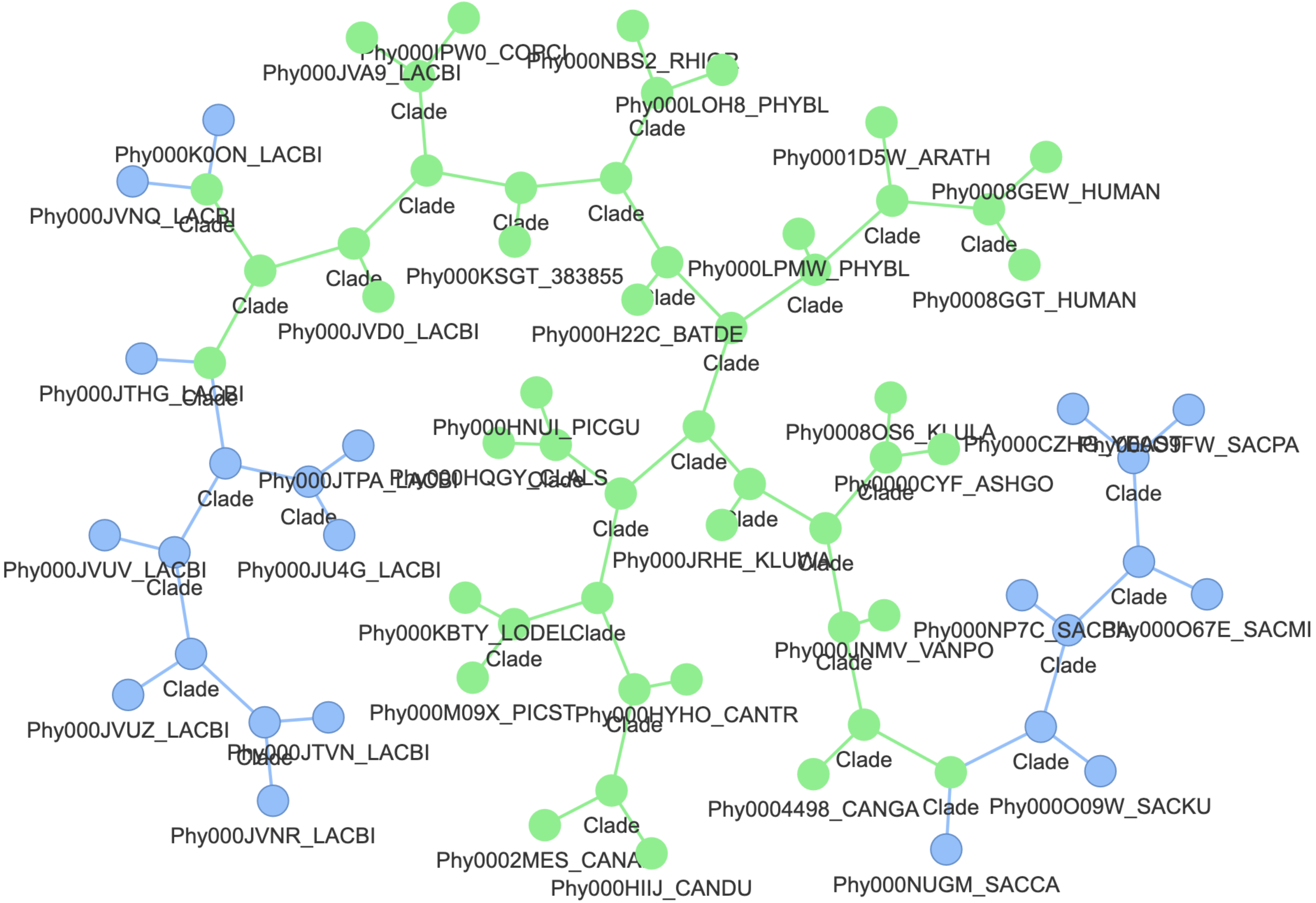
RESULTS

Table 2: Wall-clock running time of Schäfer's implementation and MAX-DsT on the six cases of studies. We denote 'timeouts' and 'not applicable' by '—' and 'n/a', respectively.

Case	Row	Graph	Running time in seconds: $t_{\text{Schäfer}}$	Running time in seconds: $t_{\text{MAX-DsT}}$	$\frac{t_{\text{Schäfer}}}{t_{\text{MAX-DsT}}}$	Row	Graph	Running time in seconds: $t_{\text{Schäfer}}$	Running time in seconds: $t_{\text{MAX-DsT}}$	$\frac{t_{\text{Schäfer}}}{t_{\text{MAX-DsT}}}$	Case
1	1	$\mathcal{T}_{22\_16}$	132.4492	60.8545	2.1	14	$\mathcal{T}_{Ph}$	719.4773	185.4435	3.8	5
2	2	$T_D,\ s=4$	109.9832	1.8577	59.2	15	$T_{0.3e6},\ s=10$	1 022.6843	9.4978	107.7	6
2	3	$T_D,\ s=5$	116.5945	1.5734	74.1	16	$T_{0.3e6},\ s=100$	1 439.42	17.3046	83.1	6
3	4	$T_L,\ s=10$	1.5289	0.5584	2.7	17	$T_{0.3e6},\ s=500$	5 989.8246	74.7689	80.1	6
3	5	$T_L,\ s=100$	12.6688	4.8969	2.5	18	$T_{0.5e6},\ s=10$	2 945.4381	15.6059	188.7	6
3	6	$T_L,\ s=1\,000$	602.9626	45.2492	13.3	19	$T_{0.5e6},\ s=100$	3 654.6010	44.1117	82.8	6
3	7	$T_L,\ s=10\,000$	19 859.2017	241.7339	82.1	20	$T_{0.5e6},\ s=500$	—	2 661.9666	n/a	6
4	8	$T_B,\ s=5$	117.9299	2.0445	57.6	21	$T_{0.75e6},\ s=10$	6 436.4398	22.7611	282.7	6
4	9	$T_B,\ s=10$	121.7708	1.9870	61.2	22	$T_{0.75e6},\ s=100$	7 807.3684	94.7391	82.4	6
4	10	$T_B,\ s=15$	125.8261	1.9811	63.5	23	$T_{0.75e6},\ s=500$	—	5 657.6981	n/a	6
4	11	$T_B,\ s=20$	125.4565	1.9592	64.0	24	$T_{1e6},\ s=10$	11 529.7250	31.8786	361.6	6
4	12	$T_B,\ s=25$	126.7537	1.9809	63.9	25	$T_{1e6},\ s=100$	14 167.638	198.9311	71.2	6
4	13	$T_B,\ s=30$	148.4265	1.9831	74.8	26	$T_{1e6},\ s=500$	—	11 422.2738	n/a	6

https://www.cs.rhul.ac.uk/home/uhac208/RP_2022/Experiments.html

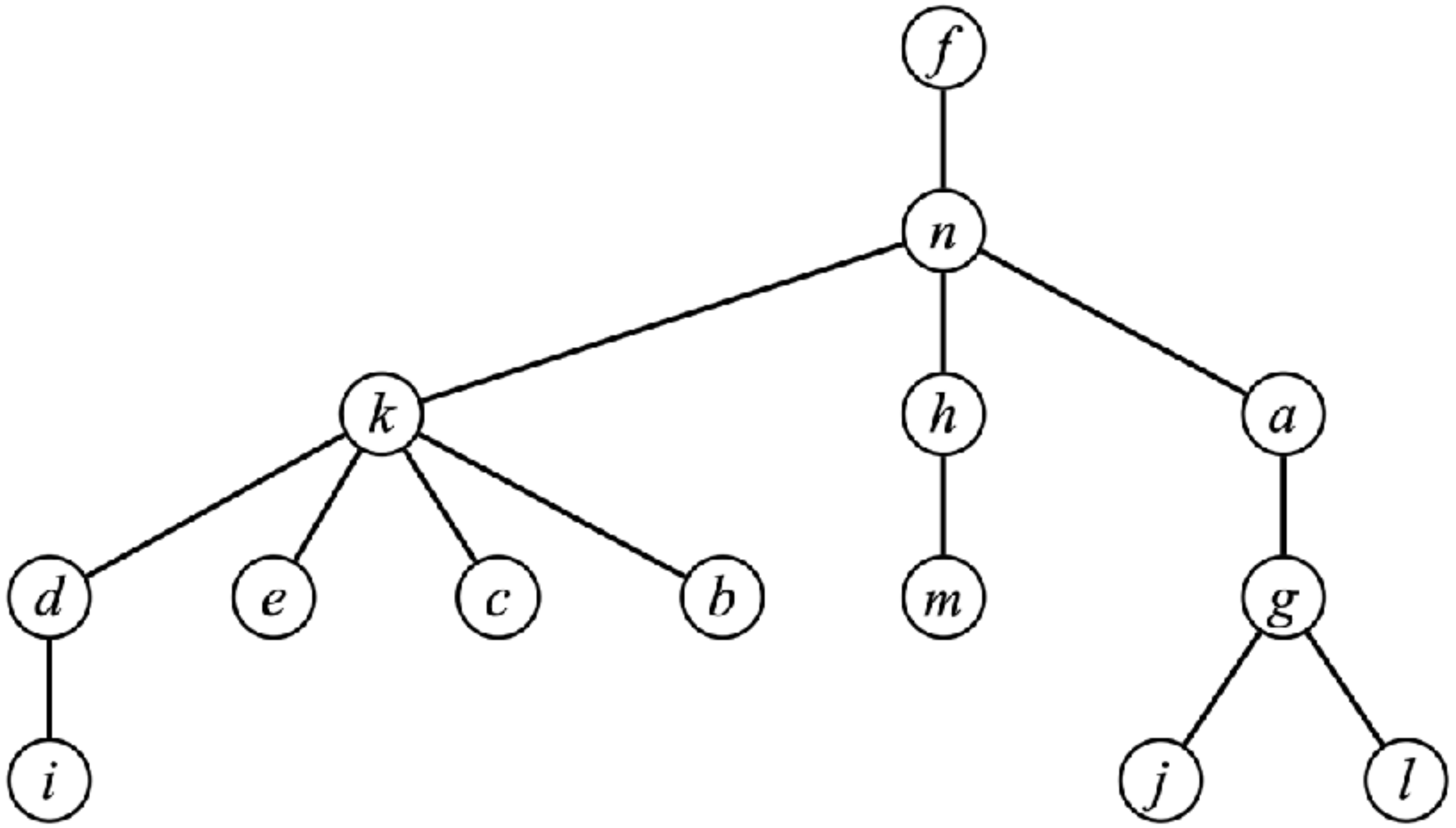
A PHYLOGENETIC TREE



It can potentially help us find relationships that we didn't see before!

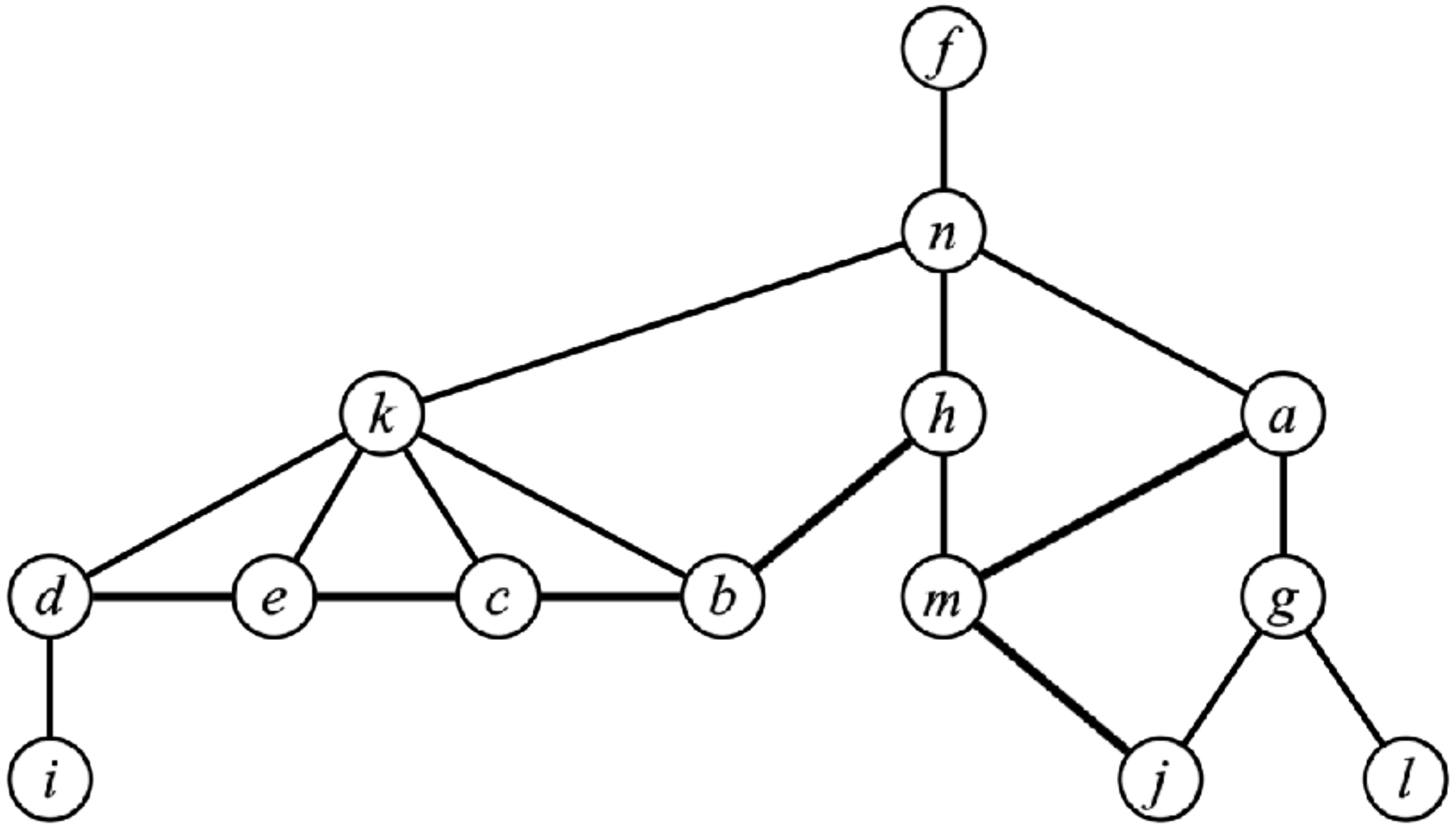
https://www.cs.rhul.ac.uk/home/uhac208/RP_2022/Database.html

FUTURE WORK



- Extend this dynamic programming approach for a broader graph family
- Preferably one with an underlying structure close to a tree
  - cordal graphs
  - outerplanar graphs
  - bounded treewidth

FUTURE WORK



- Extend this dynamic programming approach for a broader graph family
- Preferably one with an underlying structure close to a tree
  - cordal graphs
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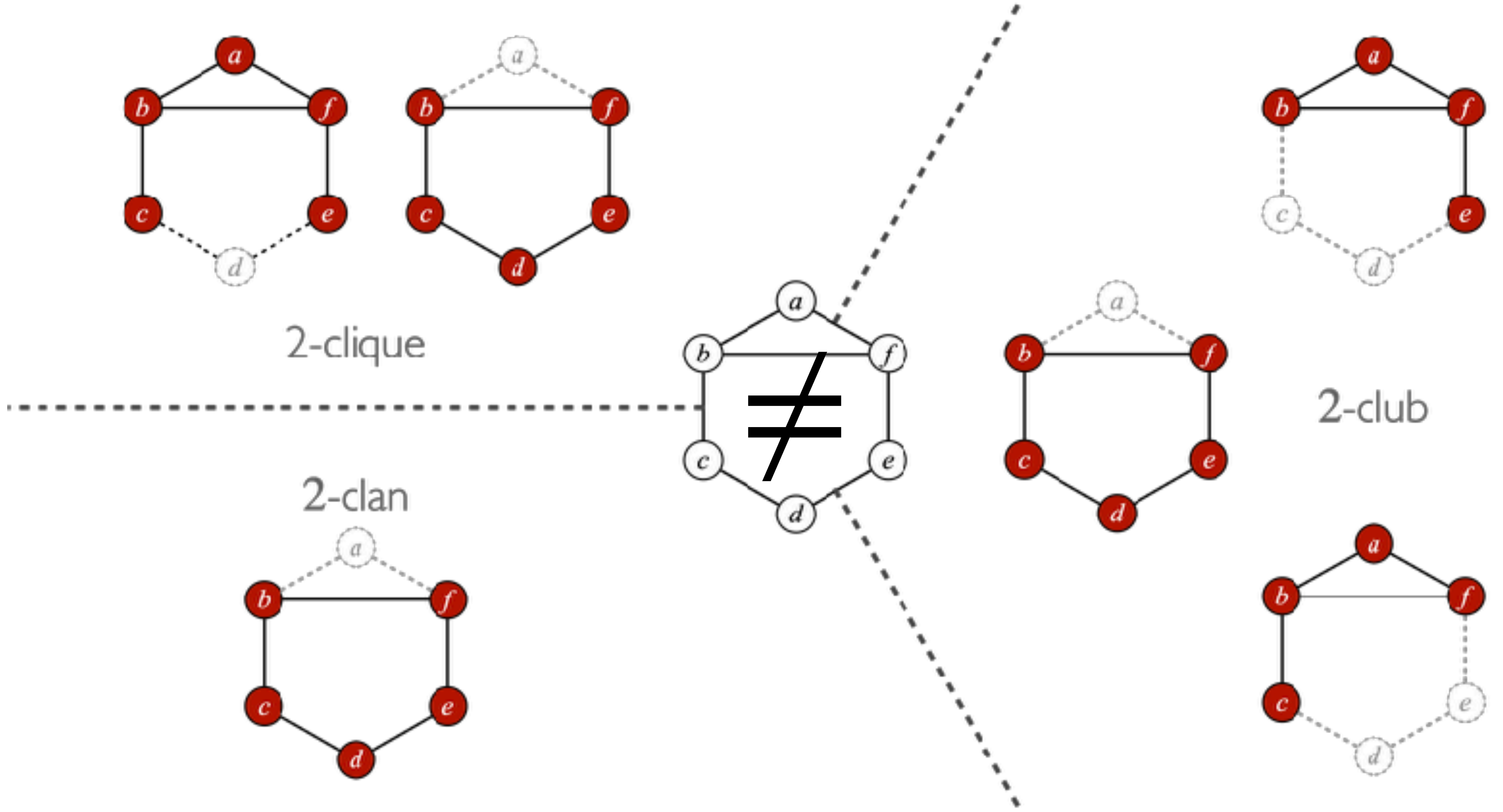
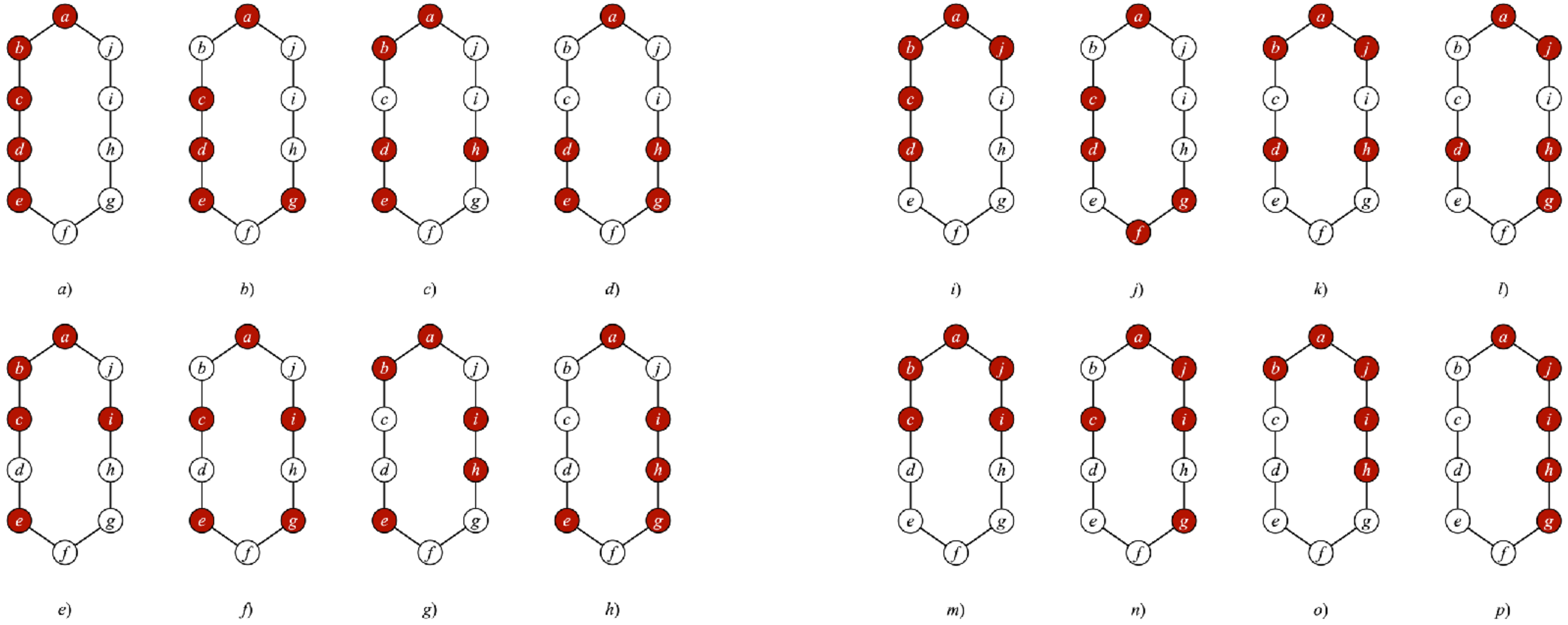
  - bounded treewidth

CHALLENGES + ADDITIONAL INFORMATION

- There could exist more than one path between some pair of vertices
- Cycle-breaking strategy or some hierarchy-like structure to avoid overlapping computations
- On other graphs, the s-clique, s-clan, and s-club might not reflect equivalent problems.

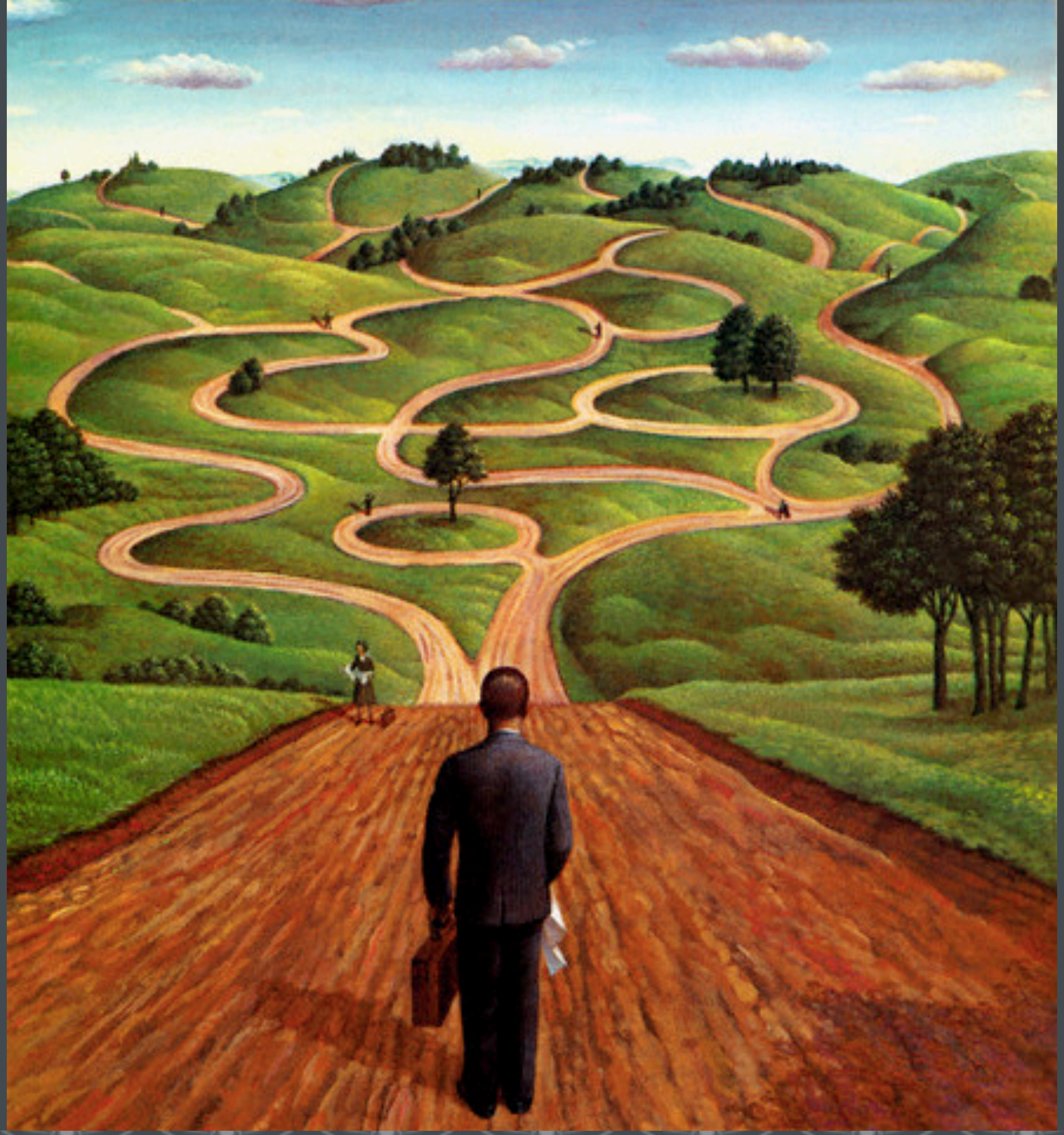


Example



$\bullet \in$ set

$dist(\bullet, \bullet) = 4$



THANK YOU

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Thank you

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