RP 2020

Unambiguity and Fewness for Nonuniform Families of Polynomial-Size Nondeterministic Finite Automata

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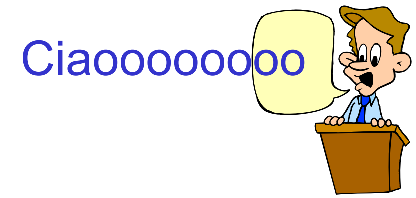
Synopsis of Today's Talk



- This seminal talk is concerned with
  - the number of accepting computation paths of nondeterministic finite automata.
- I will introduce
  - new nonuniform language families.
- I will prove
  - inclusions and separations of those nonouniform language families.

- Web Page ↪ http://tomoyukiyamakami.info
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I. Historical Background

1. Number of Accepting Computation Paths
2. Nonuniform State Complexity
3. Nondeterministic Finite Automata
4. State Complexity of Automata Families
5. Families of Promise Decision Problems
6. Nonuniform State Complexity Classes

Number of Accepting Computation Paths I

- In nondeterministic computation, the number of accepting computation paths is of interest.
- There are well-known complexity classes associated with such a number.

- ambiguous (or unique) computation
  - UP : Valiant (1976)
- few (accepting path) computation
  - FewP : Allender (1986), Allender-Rubinstein (1987)

- $\mathrm{P} \subseteq \mathrm{UP} \subseteq \mathrm{FewP} \subseteq \mathrm{NP}$

Number of Accepting Computation Paths II

- UL and FewL are later introduced by Buntrock, Jenner, Lange, and Rossmanith (1991).
- $L \subseteq UL \subseteq FewL \subseteq NL$
- Reinhardt and Allender (2000) proved that

  UL/poly = NL/poly,

  where "poly" refers to the Karp-Lipton-type polynomial-size advice.
- Bourke, Tewari, and Vinodchandran (2009) and Pavan, Tewari, and Vinodchandran (2012) introduced:

  ReachUL, ReachFewL, ReachLFew, and FewUL.

Nonuniform State Complexity

Nonuniform State Complexity

- Berman-Lingas (1977) and Sakoda-Sipser (1978) considered complexity classes of families of promise decision problems solved by nonuniform families of $n^{O(1)}$-state finite automata.
- Lately, Kapoutsis (2009,2012,2014) and Kapoutsis-Pighizzini (2015) revitalized the study.
- Yamakami (2018) further expanded the scope of the study by introducing various complexity classes.
- Yamakami (2019) presented a further work on relativizations of nonuniform state complexity classes.

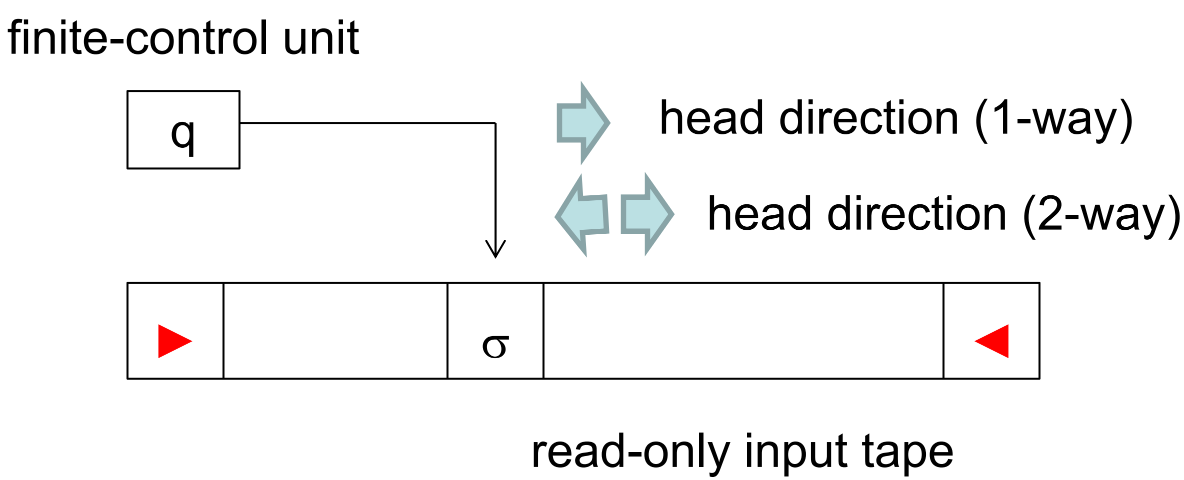
Nondeterministic Finite Automata

- In this work, we use 1-way nondeterministic finite automata (1dfa's) and 2-way nondeterministic finite automata (2dfa's) as machine models.
- 1npda: $M = (Q,\Sigma,\{\triangleright,\triangleleft\},\delta,q_0,Q_{acc},Q_{rej})$

$Q_{halt} = Q_{acc} \cup Q_{rej}$

- transition function: $\delta: (Q-Q_{halt}) \times (\Sigma \cup \{\lambda,\triangleright,\triangleleft\}) \times \Gamma \rightarrow \wp(Q)$

power set



State Complexity of an Automata Family

- For a finite automaton $M = (Q,\Sigma,\{\triangleright,\triangleleft\},q_0,Q_{acc},Q_{rej})$, the state complexity of M is $st(M) = |Q|$ (the number of inner states).
- We consider a family $\{M_n\}_{n\in N}$ of finite automata of the same type, each M_n of which is of the form $(Q_n,\Sigma_n,\{\triangleright,\triangleleft\},q_{0,n},Q_{acc,n},Q_{rej,n})$, where $\Sigma_n = \Sigma$ (same alphabet) for all $n\in N$.
- The state complexity of this family $\{M_n\}_{n\in N}$ is a function $st(n) = |Q_n|$ in length n.
- Two families $\{M_n\}_{n\in N}$ and $\{N_n\}_{n\in N}$ of finite automata are said to be equivalent if, for any $n\in N$, M_n agrees with N_n on all inputs.

Families of Promise Decision Problems

- A promise decision problem over alphabet Σ is a pair (A,R) with A,R,$\subseteq\Sigma^*$ and $A\cap R = \varnothing$, where A is a set of accepted strings (or YES instances) and R is a set of rejected strings (or NO instances).

> A string is promised if it belongs to $A\cup R$.

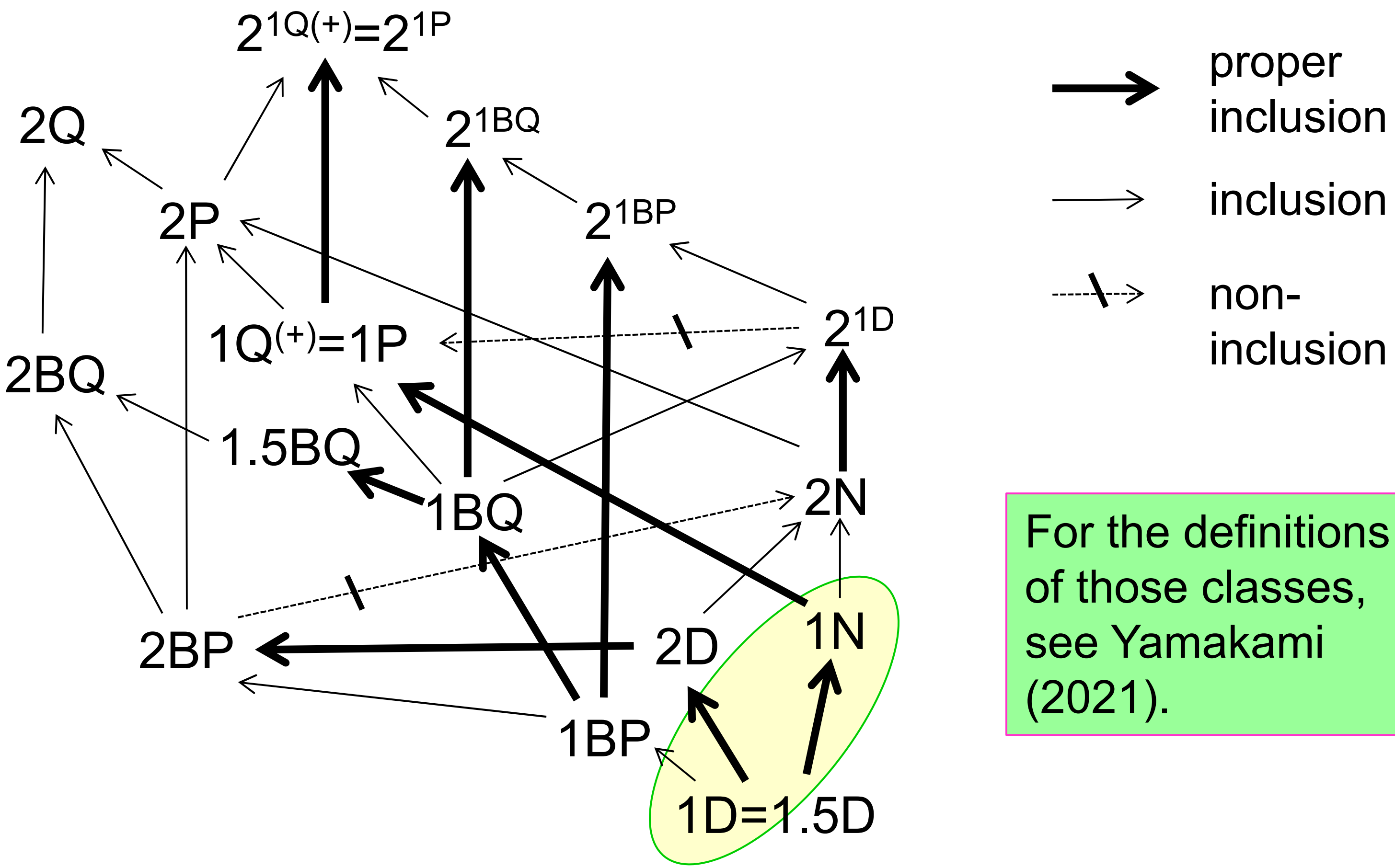
- A machine M solves (A,R) if, for all $x\in$ … and for all $x\in R$, M rejects x. However, we do not care if strings x are not promised.
- Here, we consider a family $\Lambda = \{(L_n^{(+)},L_n^{(-)})\}_{n\in N}$ of promise decision problems.
- A family $M = \{M_n\}_{n\in N}$ of finite automata solves Λ if, for each index $n\in N$, M_n solves $(L_n^{(+)},L_n^{(-)})$.

Nonuniform State Complexity Classes

- one-way case
  - 1D = set of families $\{(L_n^{(+)},L_n^{(-)})\}_{n\in N}$ of promise decision problems s.t. $\exists$ $\{M_n\}_{n\in N}$: family of poly(n)-state 1dfa's for which $\forall n\in N$ [M_n solves $(L_n^{(+)},L_n^{(-)})$ on all promised inputs].
  - 1N = ... by families of poly(n)-state 1nfa's ...
- two-way case
  - 2D = ... by families of poly(n)-state 2dfa's
  - 2N = ... by families of poly(n)-state 2nfa's

Known Inclusions and Separations

- The diagram below shows known inclusions and separations among nonuniform state complexity classes.



For the definitions of those classes, see Yamakami (2021).

II. Accepting Computation Paths

1. Unambiguity
2. Fewness
3. New Complexity Classes
4. Examples of Promise Problem Families
5. Ceilings

Unambiguity

- Let $M = \{M_n\}_{n \in N}$ denote any family of nondeterministic finite automata.

- M is unambiguous $\Leftrightarrow$ $\forall n \in N$ $\forall x \in \Sigma^{(n)}$ [there is at most one accepting computation path of M_n on input x]
- M is weak-unambiguous $\Leftrightarrow$ $\forall n \in N$ $\forall x \in \Sigma^{(n)}$ $\forall$conf: "accepting" configuration of M_n on x [there is at most one computation path of M_n on x from $conf_0$ to conf]
- M is reach-unambiguous $\Leftrightarrow$ $\forall n \in N$ $\forall x \in \Sigma^{(n)}$ $\forall$conf: configuration of M_n [there is at most one computation path of M_n on x from $conf_0$ to conf]

- Here, $\Sigma^{(n)}$ denotes the set of all promised instances in $L_n^{(+)} \cup L_n^{(-)}$.

Fewness

- Let $M = \{M_n\}_{n \in N}$ denote any family of nondeterministic finite automata.

- M is accept-few $\Leftrightarrow$ $\exists p$: polynomial $\forall n \in N$ $\forall x \in \Sigma^{(n)}$ [there are at most $p(n,|x|)$ accepting computation paths of M_n on input x]
- M is reach-few $\Leftrightarrow$ $\exists p$: polynomial $\forall n \in N$ $\forall x \in \Sigma^{(n)}$ $\forall$conf: configuration of M_n on x [there are at most $p(n,|x|)$ computation paths of M_n on x from $conf_0$ to conf]

New Complexity Classes

- we introduce six complexity classes.
  - 1Few = collection of families of promise problems solvable by families of accept-few 1nfa's with polynomially many inner states
  - 1ReachFew = 1Few whose underlying 1nfa's are reach-few
  - 1ReachFewU = 1ReachFew whose underlying 1nfa's are unambiguous
  - 1ReachU = 1ReachFewU whose underlying 1nfa's are reach-unambiguous
  - 1FewU = 1ReachU whose underlying 1nfa's are weak-unambiguous
  - 1U = 1FewU whose underlying 1nfa's are unambiguous
- Similarly, we can define their 2-way versions.

Examples of Promise Problem Families I

- We see some examples of families of promise problems.
- Let $[n] = \{ 1,2,\ldots,n \}$.
- Let $i_1,i_2,\ldots,i_k \in N^+$ and set
$$[i_1, i_2, \cdots, i_k] = 1^{i_1}01^{i_2}0 \cdots 01^{i_k}.$$

This k is called the size.

- If $r = [i_1,i_2,\ldots,i_k]$, then $(r)_{(e)}$ = the e-th entry of r.
- Let A_n = set of all strings of the form $[i_1,i_2,\ldots,i_k]$ with $i_1,i_2,\ldots,i_k \in [n]$.
- Let $A_n(k) = \{ r \in A_n \mid \text{size of } r \text{ is } k \}$.

Examples of Promise Problem Families II

- We see some examples of families of promise problems.
- Consider a family $\Lambda_1 = \{(L_n^{(+)}, L_n^{(-)})\}_{n \in N}$ with
  - $L_n^{(+)} = \{\ r_1\#r_2 \mid r_1, r_2 \in A_n(n),\ \exists! e \in [n]\ [\ (r_1)_{(e)} \neq (r_2)_{(e)}\ ]\ \}$, and
  - $L_n^{(-)} = \{\ r_1\#r_2 \mid r_1, r_2 \in A_n(n),\ \forall e \in [n]\ [\ (r_1)_{(e)} = (r_2)_{(e)}\ ]\ \}$.
- Λ_1 belongs to 1ReachU.
- Consider a family $\Lambda_2 = \{(L_n^{(+)}, L_n^{(-)})\}_{n \in N}$ with
  - $L_n^{(+)} = \{\ r_1\#r_2 \mid r_1, r_2 \in A_n(n),\ \exists e \in [n]\ [\ (r_1)_{(e)} \neq (r_2)_{(e)}\ ]\ \}$, and
  - $L_n^{(-)} = \{\ r_1\#r_2 \mid r_1, r_2 \in A_n(n),\ \forall e \in [n]\ [\ (r_1)_{(e)} = (r_2)_{(e)}\ ]\ \}$.
- Λ_2 belongs to 1FewU.

Ceilings

- We place a restriction on the size of instances.
- Let p: $\Sigma^* \rightarrow N$ be any function.
- A family Λ has a p(n)-ceiling if

  $$\forall\, n \in N\ [\, L_n^{(+)} \cup L_n^{(-)} \subseteq \Sigma^{\leq p(n)}\,]$$

set of all strings of length $\leq$ p(n)

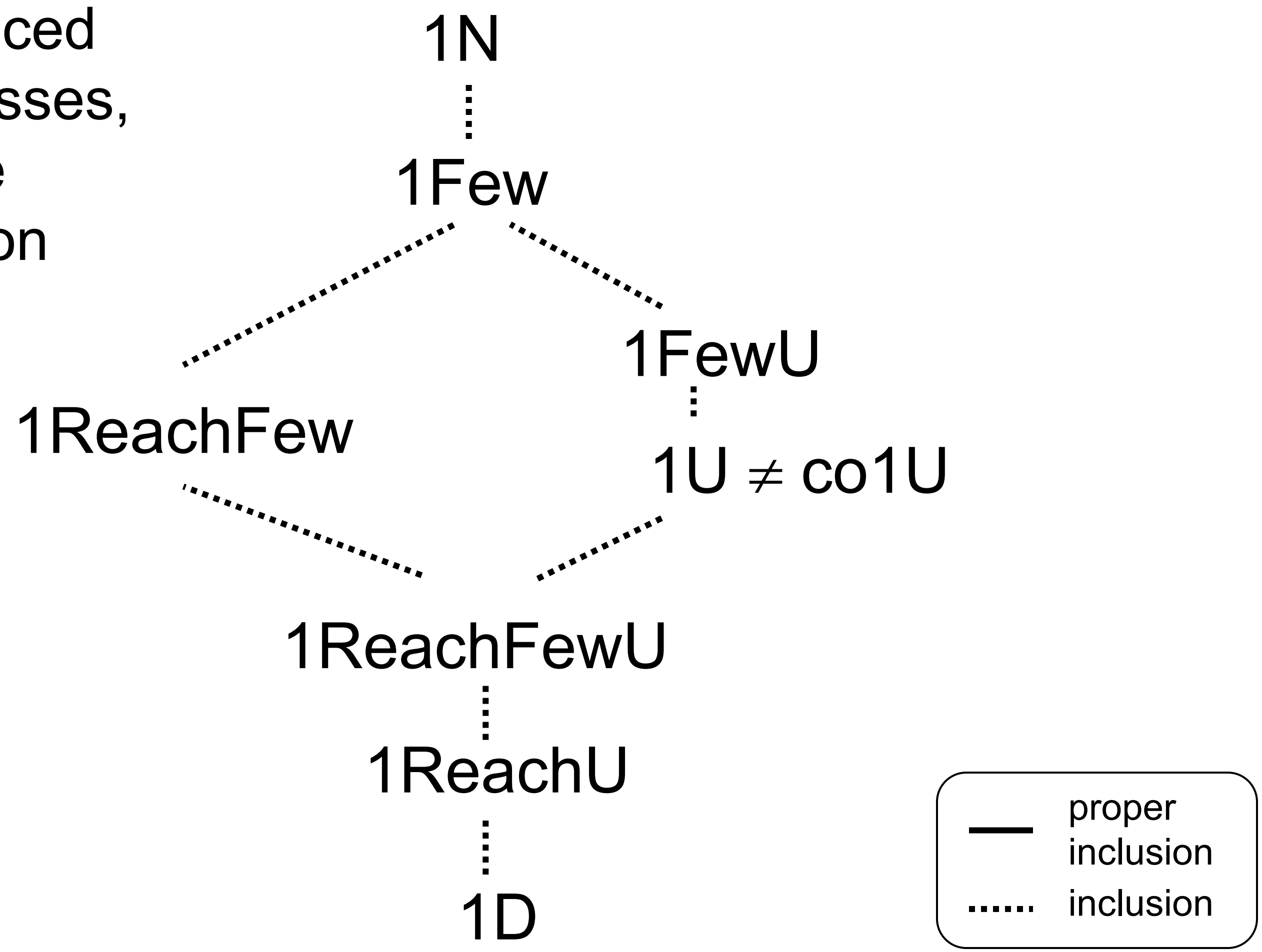
- abbreviations
  - log = collection of logarithmic functions
  - poly = ... of polynomials
  - exp = ... of exponentials
  - supexp = ... of super exponentials
- We define the following complexity classes.
  - 2D/poly = 2D restricted to polynomial ceilings
  - 2N/poly = 2N restricted to polynomial ceilings

III. Main Results

1. One-Way Case
2. New Results
3. Two-Way Case
4. New Results - Polynomial Ceiling
5. Case of Other Ceilings

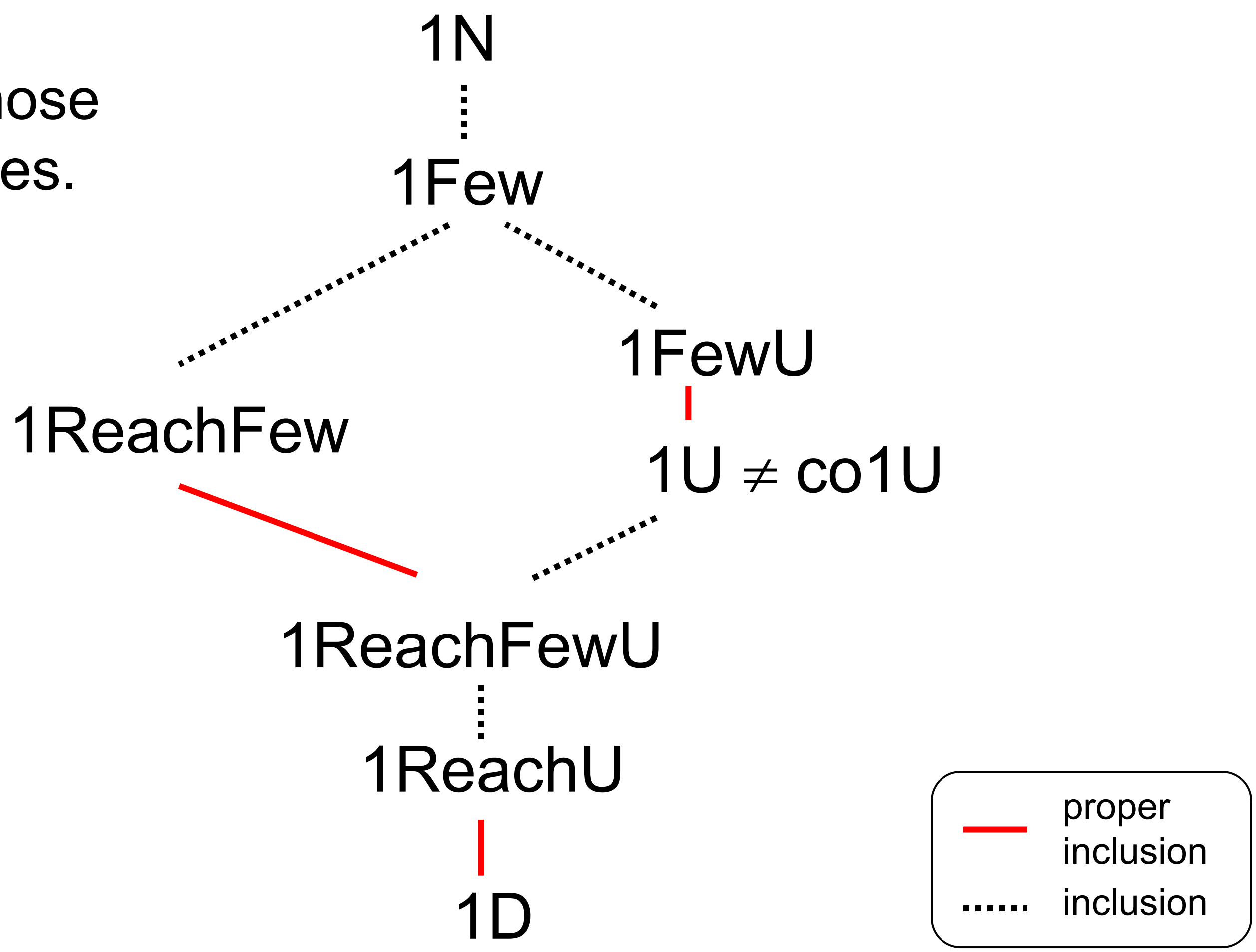
One-Way Case

We have introduced 6 complexity classes, which satisfy the following inclusion relationships.



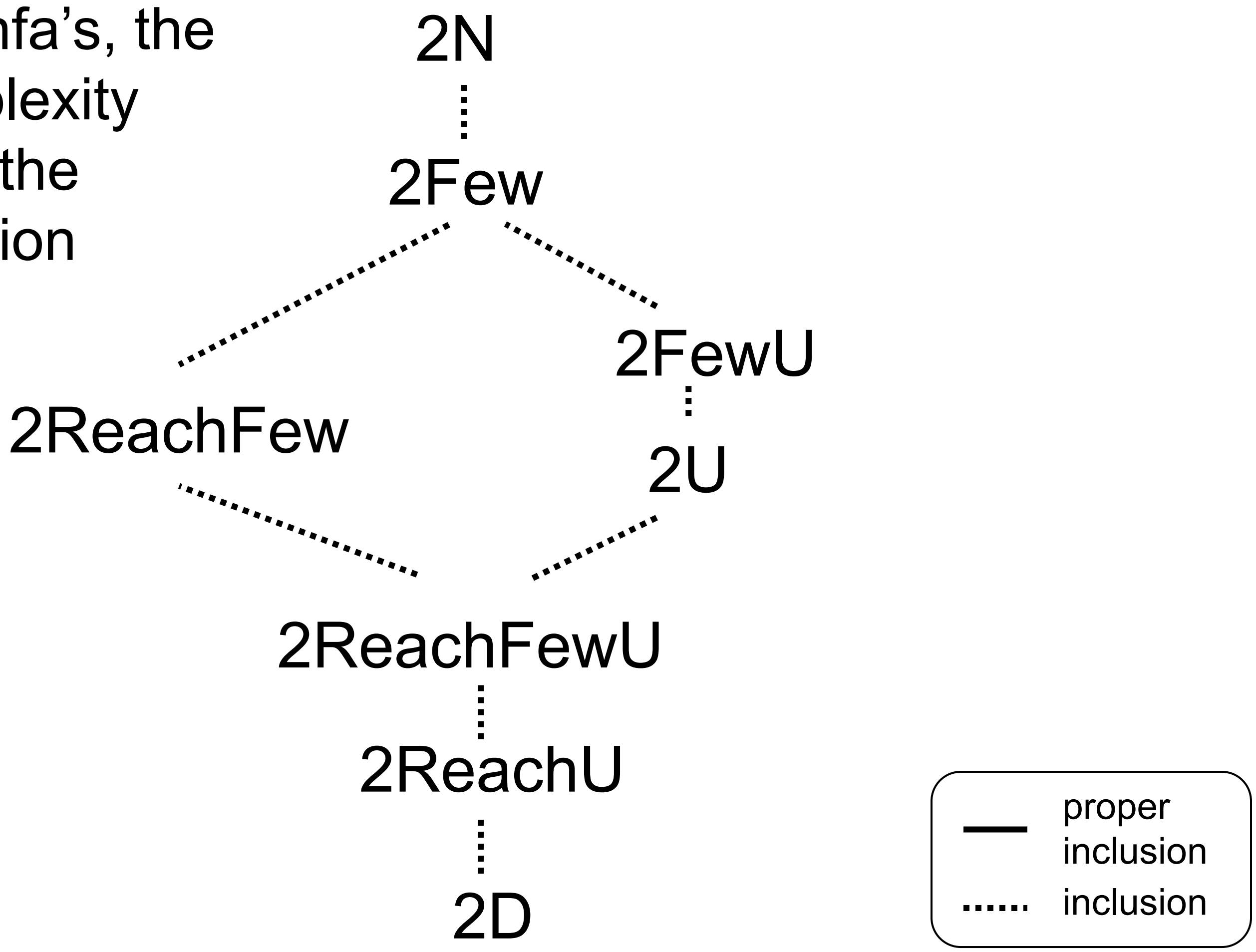
New Results

We obtain 3 separations of those complexity classes.



Two-Way Case

As for 2-way 1nfa's, the defined 6 complexity classes satisfy the following inclusion relationships.



New Results - Polynomial Ceiling

2N/poly = 2Few/poly = 2U/poly

2ReachFew/poly

2ReachFewU/poly

2ReachU/poly

2D/poly

When underlying 2-way 1nfa's have polynomial ceilings, the defined 6 complexity classes satisfy the following inclusion relationships.

— proper inclusion

...... inclusion

Case of Other Celings

- Here are additional results for the two-way case.
- log ceiling
  - 2N/log = 2D/log
- supexp ceiling
  - For any C,D$\in${ D, ReachU, ReachFewU, ReachFew, U, FewU, Few, N },

  2C/supexp $\subseteq$ 2D $\Leftrightarrow$ 2C $\subseteq$ 2D,

IV. Challenging Open Questions

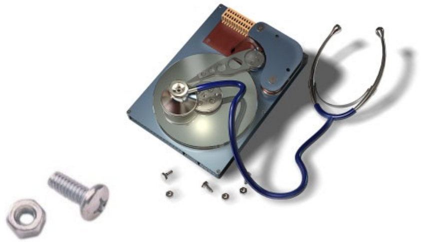
1. Challenging Open Questions

Challenging Open Questions

- There are many open problems associated with the topics of today's talk.
- Here, I list a few general questions.

1. Is it true that 1ReachFew $\subseteq$ 1FewU?
2. Is it true that 2N/poly = 2ReachFew/poly?
3. Is it true that 2N $\subseteq$ 2DPD?

2DPD = pushdown version of 2D



Thank you for listening

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I’m happy to take your question!

END